All Functions are Continuous! A Provocative Introduction to Constructive Analysis

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Abstract

Hilbert once quipped that "Taking the principle of excluded middle from the mathematician would be the same, say, as proscribing the telescope to the astronomer". But Le Verrier discovered Neptune without even looking out the window! The aim of this talk is to showcase constructive mathematics to see how far we can go without excluded middle, and hopefully discover some beautiful (or traumatising) new landscapes along the way. We begin by ironing out some misconceptions about constructivism and discussing some motivations behind it. We then present some basic analysis with examples of constructive proofs and definitions, as well as negative pathologies, such as the failure of the Intermediate Value Theorem. Finally, we venture into the land of choice sequences and provide a (surprisingly elementary) proof of Brouwer's infamous Continuity Theorem: all real-valued functions on the interval [0,1] are continuous.

Time: 3 p.m. – 4 p.m., 28th October 2020 **Location:** W.O.M.P.S. (M.S. Teams)

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