Homogenisation of Deterministic Fast-Slow Systems

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Abstract

Homogenisation of deterministic fast-slow systems is an area of some interest to applied mathematicians. For $\varepsilon > 0$, consider a system of ODEs on $\mathbb{R}^d \times M$ of the form

$$\frac{\mathrm{d}x^{(\varepsilon)}}{\mathrm{d}t} = a(x^{(\varepsilon)}, y^{(\varepsilon)}) + \frac{1}{\varepsilon} b(x^{(\varepsilon)}, y^{(\varepsilon)}) \quad \text{(slow)} \qquad \text{and} \qquad \frac{\mathrm{d}y^{(\varepsilon)}}{\mathrm{d}t} = \frac{1}{\varepsilon^2} g(y^{(\varepsilon)}) \quad \text{(fast)},$$

where $y^{(1)}$ is a 'chaotic' flow. The initial condition $y^{(\varepsilon)}(0)$ is picked randomly, with the rest of the system being deterministic. As $\varepsilon \to 0$, the slow dynamics $x^{(\varepsilon)}$ converges in distribution to the solution of a stochastic differential equation. In the first part of our talk we motivate this problem and discuss how it relates to showing a statistical limit law for $y^{(1)}$.

We then look at a discrete-time analogue of this problem. Time permitting, we prove that the limiting stochastic differential equation for the slow dynamics can be very general, even if we only consider very simple fast dynamics.

No knowledge of stochastic calculus or dynamical systems will be assumed.

Time: 3 p.m. – 4 p.m., 18^{th} November 2020 **Location:** W.O.M.P.S. (M.S. Teams)

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Website: `https://warwick.ac.uk/fac/sci/maths/research/events/seminars/areas/postgraduate/20-21`