Applications of entropy compression method to graph colorings

Jan Volec

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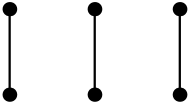

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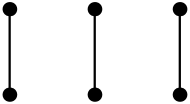

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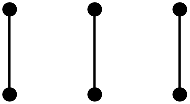

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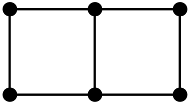

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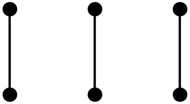

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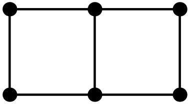

$$\mathbf{P}[\mathrm{Fail}] \leq \sum_{i=1}^{m} 1 - \mathbf{P}[P_i] = \frac{m}{k}, \text{ which is } < 1 \text{ for } k = m + 1$$

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- Example: graph G with maximum degree $\Delta \rightarrow \exists$ 4Δ-coloring
- Well, simple greedy algorithm gives $(\Delta + 1)$-coloring...

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- The method independently discovered by Schweitzer (2009)

Edge-colorings of graphs

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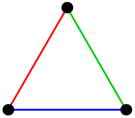
- we definitely need at least Δ colors, sometimes we need more



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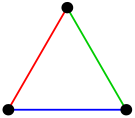

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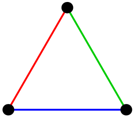

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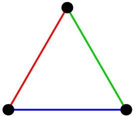

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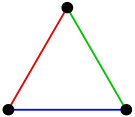

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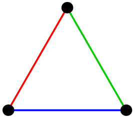

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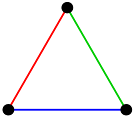

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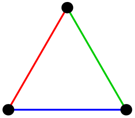

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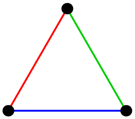

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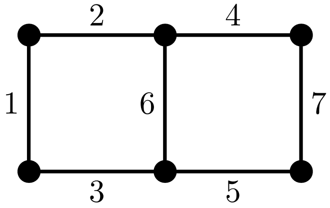

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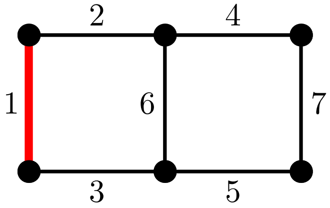
colors from $\mathcal{R}$ **R**



COLOR LOG: C
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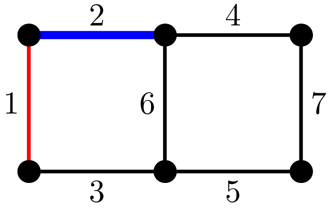
colors from $\mathcal{R}$ R **B**



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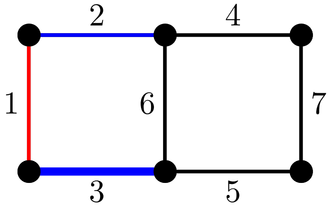

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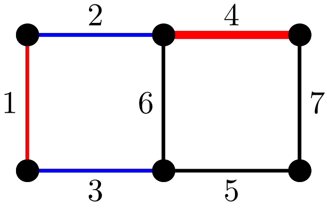

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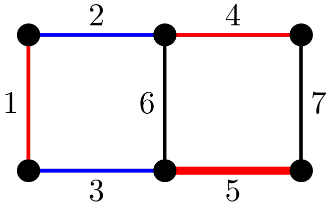

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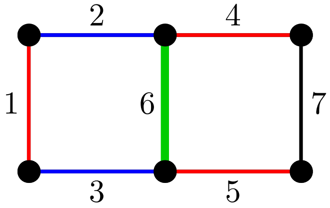
colors from $\mathcal{R}$ R B B R R **G**



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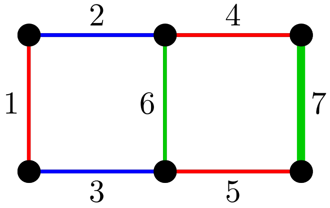

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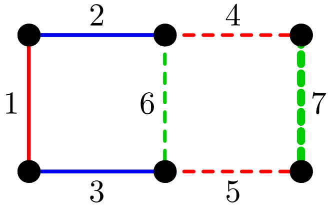

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Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

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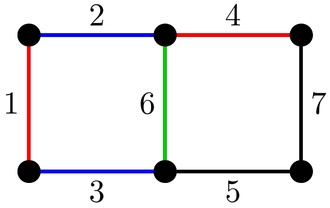
colors from $\mathcal{R}$ R B B R R G **G**

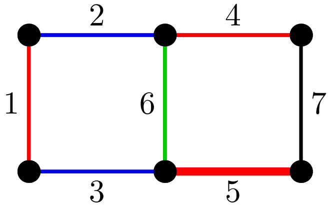


COLOR LOG: C C C C C C C **U U**
CYCLE LOG: **0 0**

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

- $e :=$ uncolored edge with minimum index
- Try to color e, does it create any 2-colored 2ℓ-cycle?
- If YES, then uncolor edges $c_3, c_4, \ldots, c_{\ell-1}$ and e
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R G G **R**

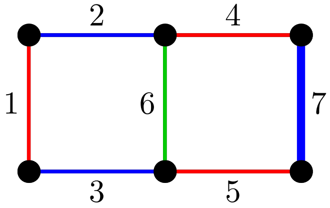


COLOR LOG: C C C C C C C U U **C**
CYCLE LOG: 0 0

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

- $e :=$ uncolored edge with minimum index
- Try to color e, does it create any 2-colored 2ℓ-cycle?
- If YES, then uncolor edges $c_3, c_4, \ldots, c_{\ell-1}$ and e
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R G G R **B**

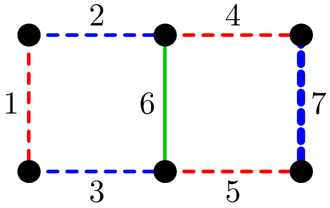


COLOR LOG: C C C C C C C U U C **C**
CYCLE LOG: 0 0

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

- $e :=$ uncolored edge with minimum index
- Try to color e, does it create any 2-colored 2ℓ-cycle?
- If YES, then uncolor edges $c_3, c_4, \dots, c_{\ell-1}$ and e
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R G G R **B**



COLOR LOG: C C C C C C C U U C C
CYCLE LOG: 0 0

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

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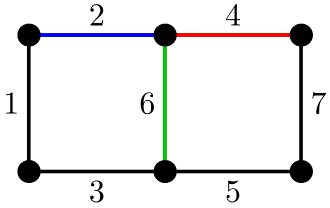
colors from $\mathcal{R}$ R B B R R G G R **B**



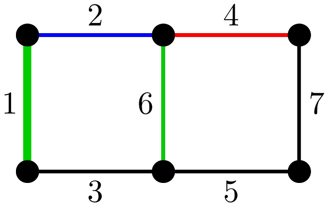
COLOR LOG: C C C C C C C U U C C **U U U U**

CYCLE LOG: 0 0 **0 0 0 0**

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

- $e :=$ uncolored edge with minimum index
- Try to color e, does it create any 2-colored 2ℓ-cycle?
- If YES, then uncolor edges $c_3, c_4, \ldots, c_{\ell-1}$ and e
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R G G R B **G**



COLOR LOG: C C C C C C C U U C C U U U U **C**
CYCLE LOG: 0 0 0 0 0 0

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

- $e :=$ uncolored edge with minimum index
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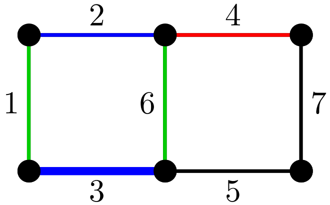
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R G G R B G **B**



COLOR LOG: C C C C C C C U U C C U U U U C **C**
CYCLE LOG: 0 0 0 0 0 0

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

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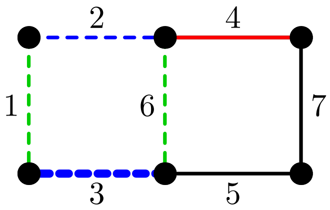
colors from $\mathcal{R}$ R B B R R G G R B G **B**



COLOR LOG: C C C C C C C U U C C U U U U C C
CYCLE LOG: 0 0 0 0 0 0

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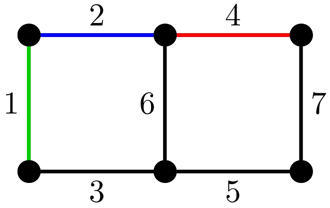
colors from $\mathcal{R}$ R B B R R G G R B G **B**

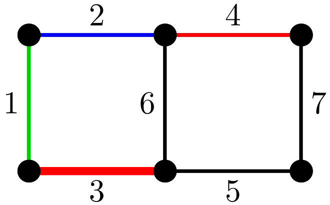


COLOR LOG: C C C C C C C U U C C U U U U C C **U U**
CYCLE LOG: 0 0 0 0 0 0 **0 0**

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

- $e :=$ uncolored edge with minimum index
- Try to color e, does it create any 2-colored 2ℓ-cycle?
- If YES, then uncolor edges $c_3, c_4, \ldots, c_{\ell-1}$ and e
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R G G R B G B **R**



COLOR LOG: C C C C C C C U U C C U U U U C C U U **C**
CYCLE LOG: 0 0 0 0 0 0 0 0

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$

- $e :=$ uncolored edge with minimum index
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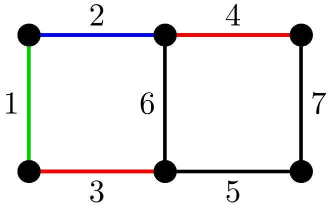
colors from $\mathcal{R}$ R B B R R G G R B G B R...



COLOR LOG: C C C C C C C U U C C U U U U C C U U C ...
CYCLE LOG: 0 0 0 0 0 0 0 0 ...

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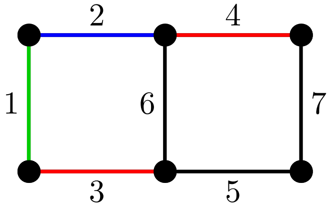
colors from $\mathcal{R}$ R B B R R G G R B G B R...



COLOR LOG: C C C C C C C U U C C U U U U C C U U C ...
CYCLE LOG: 0 0 0 0 0 0 0 0 ...

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$
Claim: COLOR and CYCLE LOGs $\longrightarrow$ current set of colored edges

- $e :=$ uncolored edge with minimum index
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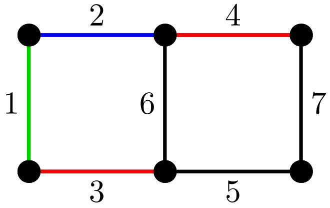
- If YES, then uncolor edges $c_3, c_4, \ldots, c_{\ell-1}$ and e
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R G G R B G B R...



COLOR LOG: C C C C C C C U U C C U U U U C C U U C ...
CYCLE LOG: 0 0 0 0 0 0 0 0 ...

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$
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Claim: current coloring and LOGs $\longrightarrow$ the whole sequence $\mathcal{R}$

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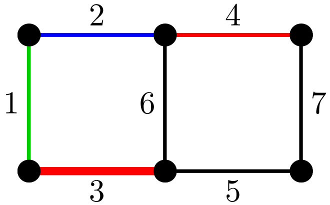
colors from $\mathcal{R}$ R B B R R G G R B G B **R** ...



COLOR LOG: C C C C C C C U U C C U U U U C C U U **C** ...
CYCLE LOG: 0 0 0 0 0 0 0 0 ...

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$
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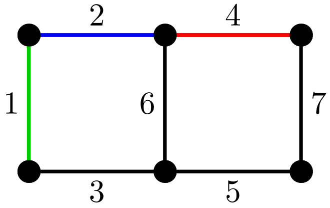
colors from $\mathcal{R}$ R B B R R G G R B G B **R** ...



COLOR LOG: C C C C C C C U U C C U U U U C C U U ~~C~~ ...
CYCLE LOG: 0 0 0 0 0 0 0 0 ...

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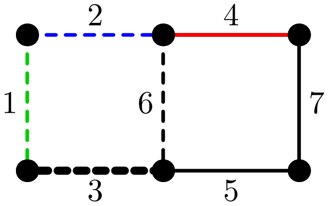
colors from $\mathcal{R}$ R B B R R G G R B G **B** R...



COLOR LOG: C C C C C C C U U C C U U U U C **C U U** ~~C~~ ...
CYCLE LOG: 0 0 0 0 0 0 **0 0** ...

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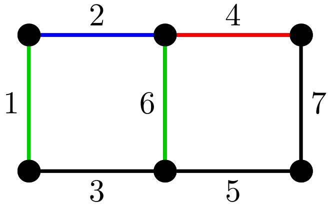
colors from $\mathcal{R}$ R B B R R G G R B G **B** R...



COLOR LOG: C C C C C C C U U C C U U U U C ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: 0 0 0 0 0 0 ~~0~~ ~~0~~ ...

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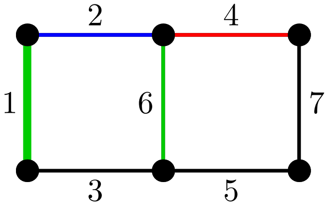
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R G G R B **G** B R...



COLOR LOG: C C C C C C C U U C C U U U U **C** ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: 0 0 0 0 0 0 ~~0~~ ~~0~~ ...

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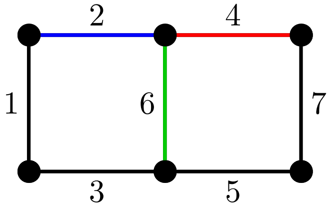
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R G G R B **G** B R...



COLOR LOG: C C C C C C C U U C C U U U U ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: 0 0 0 0 0 0 ~~0~~ ~~0~~ ...

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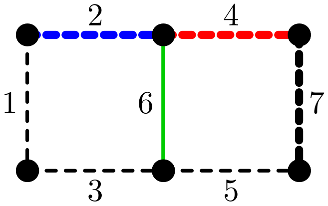
colors from $\mathcal{R}$ R B B R R G G R **B** G B R...



COLOR LOG: C C C C C C C U U C **C U U U U** ~~C C U U C~~ ...
CYCLE LOG: 0 0 **0 0 0 0** ~~0 0~~ ...

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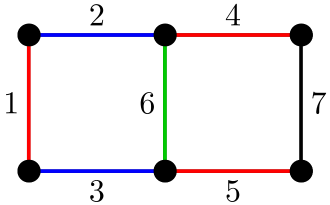
colors from $\mathcal{R}$ R B B R R G G R **B** G B R...



COLOR LOG: C C C C C C C U U C ~~C U U U U C C U U C~~ ...
CYCLE LOG: 0 0 ~~0 0 0 0 0 0~~ ...

Claim: # of 2ℓ-cycles around fixed edge e is at most $(\Delta - 1)^{2\ell-2}$
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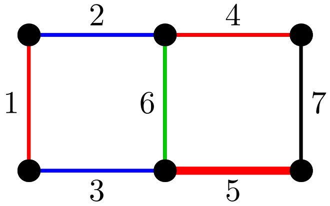
colors from $\mathcal{R}$ R B B R R G G **R** B G B R...



COLOR LOG: C C C C C C C U U **C** ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: 0 0 ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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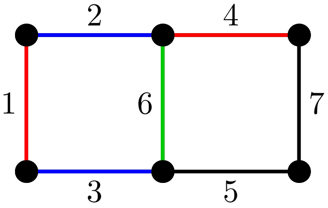
colors from $\mathcal{R}$ R B B R R G G **R** B G B R...



COLOR LOG: C C C C C C C U U ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: 0 0 ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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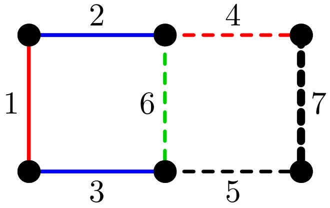
colors from $\mathcal{R}$ R B B R R G **G** R B G B R...



COLOR LOG: C C C C C C **C U U** ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: **0 0** ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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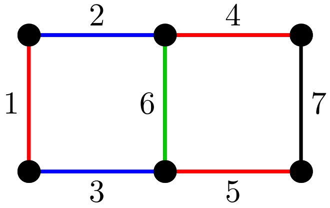
colors from $\mathcal{R}$ R B B R R G **G** R B G B R...



COLOR LOG: C C C C C C ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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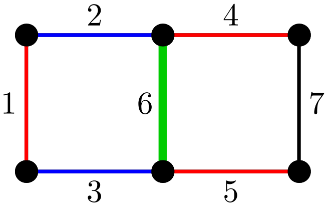
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R **G** G R B G B R...



COLOR LOG: C C C C C **C** ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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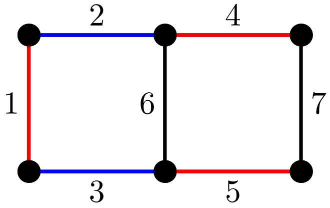
- write LOG record about this step

colors from $\mathcal{R}$ R B B R R **G** G R B G B R...



COLOR LOG: C C C C C ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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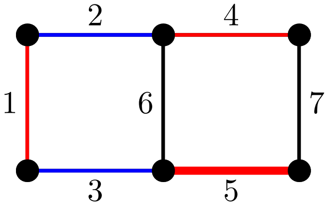

COLOR LOG: C C C C **C** ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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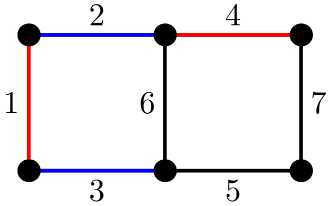

COLOR LOG: C C C C ~~C~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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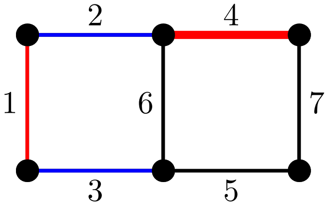

COLOR LOG: C C C **C** ~~C~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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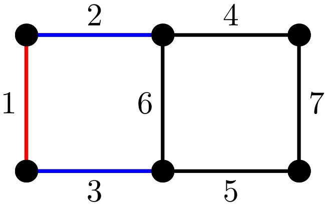

COLOR LOG: C C C ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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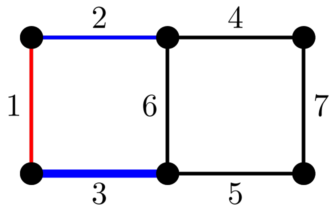
colors from $\mathcal{R}$ R B **B** R R G G R B G B R...



COLOR LOG: C C **C** ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
CYCLE LOG: ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ ...

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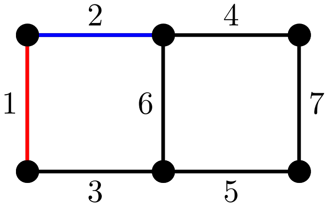

COLOR LOG: C C ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
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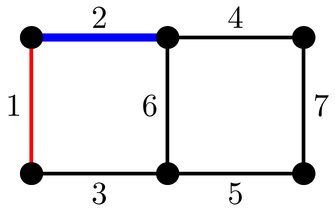

COLOR LOG: C **C** ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
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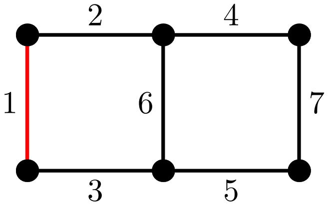

COLOR LOG: C ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
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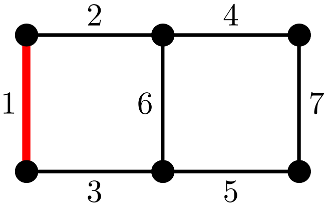

COLOR LOG: **C** ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
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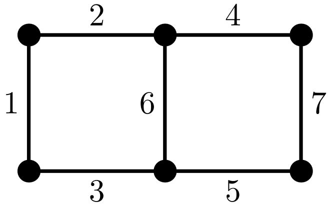

COLOR LOG: ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~U~~ ~~U~~ ~~C~~ ~~C~~ ~~U~~ ~~U~~ ~~C~~ ...
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CYCLE LOG: 0 0 0 0 0 0 0 0 ...

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There are $(4\Delta)^t$ choices for $\mathcal{R}$, so there must be a good choice!

Thank you for your attention!