

RG reading plan 2026 2027

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Introduction to a Renormalisation Group Method

Our first aim is to understand the perturbative RG map in Chapter 5, especially how Gaussian integration and localisation produce the beta-functions in §5.3. We will take turns presenting the material, with first-year PhD students in mind. The blocks below specify reading priorities rather than a fixed number of lectures.

Read means work through the argument; **skim** means retain the definitions and main statements. **Optional material** can be postponed. Section titles and numbering follow the book.

Block A Critical phenomena and effective potentials

Coverage: Introduce Gibbs measures, the thermodynamic limit and critical exponents through the Ising model. Mean-field theory then supplies a calculable example of an effective potential. Finish by introducing the Gaussian reference model and the quartic interaction that will be studied by RG.

1.2 Ising Model

Read for motivation: magnetisation, susceptibility, correlation length and critical behaviour. Keep the distinction between finite-volume analyticity and singular behaviour in the thermodynamic limit.

1.3 Spin Systems and Universality

Skim the $O(n)$ spin-system notation and generalised Laplacian needed in §1.4. The broader universality discussion provides context.

1.4 Mean-Field Model

Read all three subsections. Emphasise the auxiliary-field representation and how minimising the renormalised potential determines the limiting behaviour.

1.4.1 Critical Behaviour of the Mean-Field Model

1.4.2 Renormalised Measure

1.4.3 Magnetisation and Susceptibility: Proof of Theorem 1.4.1

1.5.1 Gaussian Free Field and 1.5.3 The Bubble Diagram

Added background. Briefly introduce the GFF covariance, then explain the bubble diagram and its logarithmic divergence in dimension four. The detailed random-walk treatment in §1.5.2 can be postponed.

1.6 $|\phi|^4$ Model

Added background. Read 1.6.1 Definition of the $|\phi|^4$ Model; skim 1.6.2 Critical Exponents of the $|\phi|^4$ Model. State the susceptibility result to explain what the later calculation is meant to establish.

Block B Gaussian calculus

Coverage: Learn to compute Gaussian expectations and connected contributions. These are the tools for expanding the effective interaction to second order in the couplings.

2.1 Gaussian Integration

Read selectively but carefully: Gaussian convolution, the covariance Laplacian and Proposition 2.1.6, which expresses convolution as the exponential of that Laplacian. Include Wick contractions, Wick ordering and the convolution semigroup property. These also prepare the comparison with continuous flow equations.

2.2 Cumulants

Read in full. Explain why taking the logarithm selects connected contributions. Work through Exercise 2.2.4: its contraction formula is used directly in the proof of the perturbative flow.

Block C Progressive integration and finite range

Coverage: Understand how a covariance decomposition becomes successive integrations over independent Gaussian fluctuation fields. Finite range gives independence of sufficiently separated regions. This principle has uses well beyond the hierarchical model; the particular Euclidean decomposition is background for our present goal.

3.1 Progressive Integration

Read in full, including Definition 3.1.1 and Exercise 3.1.2. The identity (3.1.2) is the starting point of RG, and should be explained on the board.

3.2 Finite-Range Decomposition: Continuum

Optional, with value beyond this book. Proposition 3.2.3 decomposes a power-law kernel into positive-definite contributions of finite range. Its Fourier-analysis and scaling argument is a useful separate topic.

3.3 Finite-Range Decomposition: Lattice

For the direct route, state the decomposition and omit its construction. Retain the original subsections as follows:

3.3.1 Statement of the Decomposition

Skim Proposition 3.3.1: positivity, finite range and scaling estimates. This motivates Chapter 4, but its proof is not a prerequisite.

3.3.2 Integral Decomposition

Skip for now; part of the Euclidean covariance construction.

3.3.3 Chebyshev Polynomials

Optional beyond this book: the connection with finite propagation for a discrete wave equation is useful in spectral theory.

3.3.4 Proof of Proposition 3.3.1

Skip for the route to Chapter 5.

3.4 Finite-Range Decomposition: Torus

Skip for now. The hierarchical finite-volume decomposition will be supplied independently in §4.1.

Block D The hierarchical model

Coverage. Define the model on which Chapter 5 operates. Learn the block structure, the covariance of each fluctuation field and the Gaussian representation of the interacting measure. The covariance sums are the numerical input to the beta-functions.

4.1 Hierarchical GFF

Read the construction and the properties needed for block integration. All three subsections have a role:

4.1.1 Hierarchical Fields

Read the block definitions and the structure of hierarchical fields.

4.1.2 Construction of Hierarchical GFF

Read the projections and covariance decomposition, especially Proposition 4.1.9. Include the final constant field, which handles the finite-volume mode.

4.1.3 Properties of Hierarchical Covariances

Read the covariance sums (4.1.20). Work through Exercise 4.1.11, especially the sum of squared scale covariances; Exercise 4.1.12 links these sums to the hierarchical bubble diagram.

The zero-sum identity (4.1.13) is especially important: it simplifies the second-order calculation. This is a property of the chosen hierarchical covariance and must not be assumed for a general finite-range decomposition. Exercises 4.1.8 and 4.1.13 can remain optional.

4.2 Hierarchical $|\phi|^4$ Model

Read the model definition and state Theorem 4.2.1. It gives the target susceptibility asymptotics for the hierarchical setting.

4.3 GFF and $|\phi|^4$ Model

Read the change to a massive Gaussian reference measure and Exercise 4.3.1. This identifies the initial interaction and the susceptibility in terms of the final effective functional. Include Exercise 4.3.2 as an optional discussion of the coupling defined through the truncated four-point function; its identification with the running quartic coupling uses the polynomial approximation stated there.

First target in Chapter 5

Read 5.1 Reduction to Block Analysis and 5.2.1 Local Coordinates, 5.2.2 Localisation and 5.2.3 Perturbative Map. The first milestone is to explain (5.2.17) as the localised second-order cumulant expansion. Then read 5.3.1 Statement of the Perturbative Flow and 5.3.2 Proof of the Perturbative Flow, reproducing the quartic recursion (5.3.2) and its coefficient.

Sections 5.2.4 Definition of the Renormalisation Group Map and 5.2.5 The Last Renormalisation Group Step are needed to understand the exact map and its remainder, but their details can follow the coefficient calculation. Chapter 6 then analyses iteration of the flow. The later nonperturbative estimates are still needed to prove that the exact RG is controlled by this perturbative description.