

THE CONFORMALLY INVARIANT METRIC ON CLE_4

ABSTRACT. A Conformal Loop Ensemble (CLE_κ) is a canonical family of random collections of non-crossing loops in a two-dimensional planar domain, indexed by a parameter $\kappa \in (8/3, 8)$. CLEs describe the scaling limits of interfaces in critical statistical mechanics models on planar lattices and random planar maps such as the critical Ising model ($\kappa = 3$), the critical percolation ($\kappa = 6$), and the FK-Ising model ($\kappa = 16/3$). In this talk, I will consider CLE with the parameter $\kappa = 4$, the critical value at or below which the loops are simple and do not intersect each other or the domain boundary. We show that there exists a unique metric on the set of CLE_4 loops which is conformally invariant, local, geodesic metric, and it is a measurable function of the CLE_4 . The metric is constructed as the limit of the (appropriately) renormalized graph metric on the loops of CLE_κ as $\kappa \downarrow 4$, where we say that there is an edge between two vertices corresponding to two distinct CLE_κ loops if and only if the loops intersect each other. We also establish properties of the geodesics by showing that any geodesic between two loops is supported on the CLE_4 loops (off a set of Hausdorff dimension zero) and does not intersect the domain boundary. Based on joint work with Emmanuel Kammerer, Jason Miller, and Yi Tian.