

# Risk and Uncertainty in Reservoir Modelling

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# Outline

- Oil Fields
  - What are they, what do they look like?
- Mathematics of Reservoir Simulation
  - Equations, getting it right
- Uncertainty in Reservoir Modelling
  - Where does it come from?
- Examples
  - Need for advanced computational techniques

# Developing Oil Fields



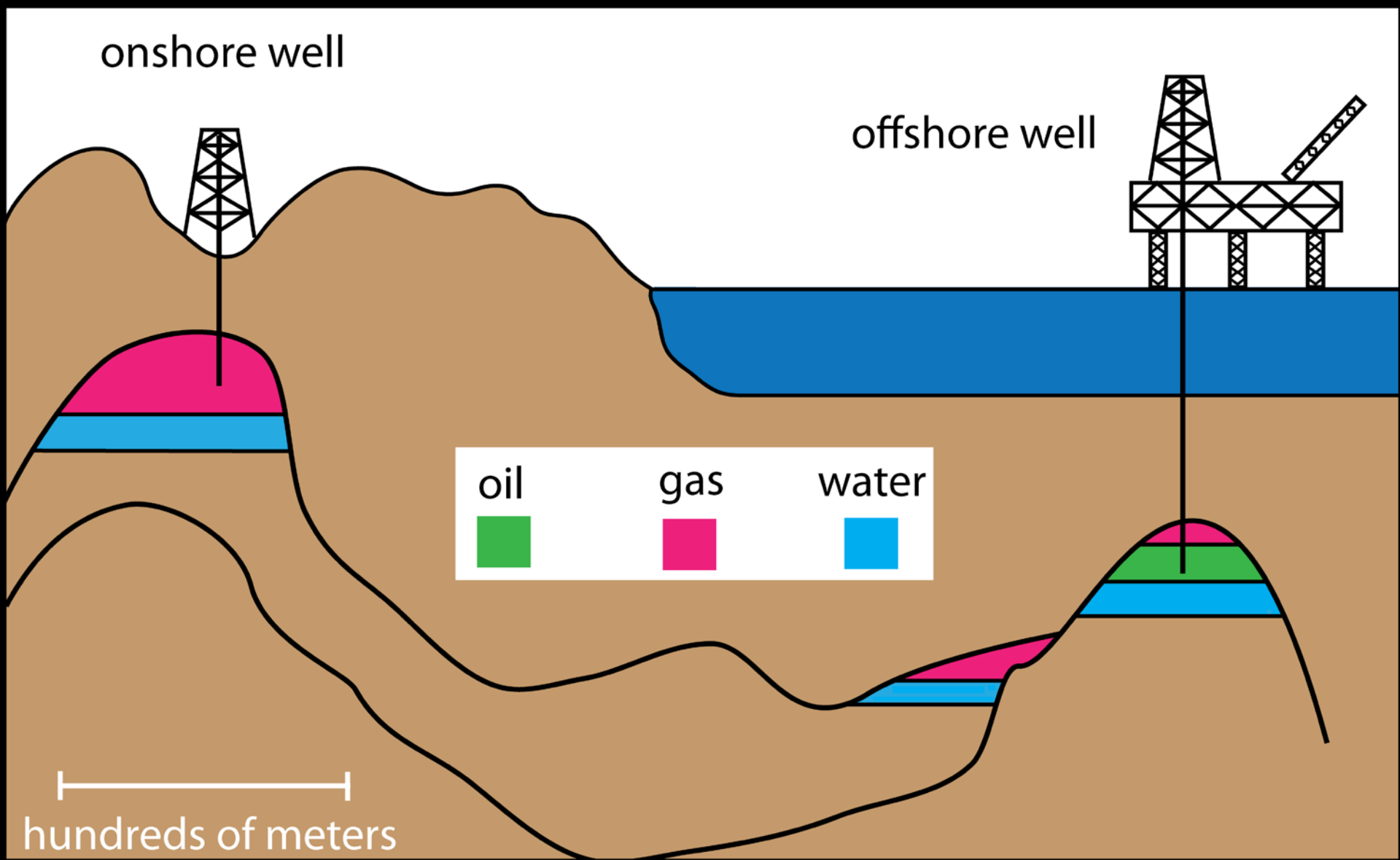
# Oil Fields



# What Does an OilField Look Like?



# Oil Reservoir



# Wytch Farm Oil Field



**TIDES £735m**

New England style colonial house built in 2003. Six bedrooms, heated outdoor pool, boat house and a 7 metre jetty onto the private waterfront

**THE BRINK \$6m**

**SOLD**

Sold in March 1999 for £1.5million. Following updating, it came back on the market selling for around £1million. Includes a hand-made kitchen and computerised lighting

**SANDBANKS YACHT COMPANY £12.75m**

Three new houses built on the site of a former yacht yard priced at £4.25 million each. They each have five bedrooms with en-suites. There is under-floor heating and iPad-controlled lighting, heating, curtains and alarms

**VELSHEDA £3.8m**

**SOLD**

One of two three-storey semi-detached houses built between 2005 and 2007. Sold at the height of the boom in August 2007 for £3.2 million. Sold again in September 2012 for £3.8 million

**TANSANEE £6.15m**

The proposed dwelling has four bedrooms, all en-suite. There will be a pool and a gym, cinema and wine cellar. All floors to be connected by lift

**STARBOARD LIGHTS £4.4m**

Built in 1995. Has four bedrooms, with hardwood bedrooms, and one of the widest waterfront plots on Sandbanks. Looks quite plain compared with nearby contemporary properties and could be a candidate for redevelopment

**GOLD COAST: The Sandbanks peninsula in Dorset**

**SOLARIS £6.25m**

New five-bedroom property with pool, Jacuzzi and cinema. Has been put up for sale after a repossession

**FLAG HOUSE £4.75m**

Arts & Crafts house which still has its original decorative cornicing and herringbone woodblock floors. The house is under offer and the planners have indicated that they do not want it demolished

**HARBOUR WATCH £7.4m**

Six bedrooms, six reception rooms and a seven-car underground garage

**AMIRI £7m**

**SALE AHEAD**

Six-bedroom house with a pool in the basement plus a gym and sauna. Developer paid £2.2 million for the site in 2002

**HARBOUR EDGE £4.25m**

**SOLD**

Until 1994, the site was occupied by a Pifine helicopter. Bought by John Lavers in 1995 for £25,000 for his Aunt Mimi

**BANKS ROAD £13.5m**

The site - sold by Savills last year for nearly £5m - has planning permission for two 5-bedroom houses. No mortgage though as the houses face onto a public beach

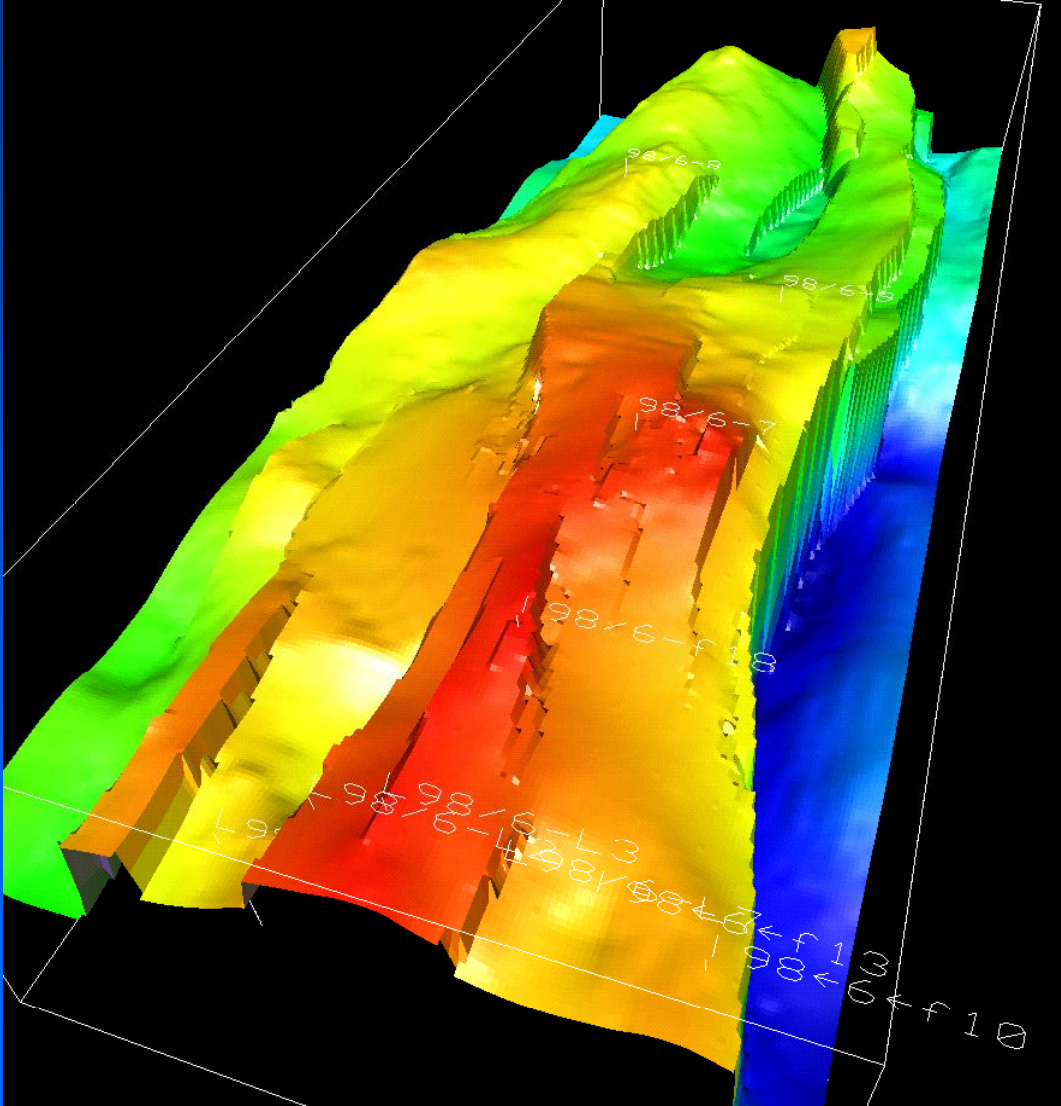
**UNDER OFFER**

# Wytch Farm from the Sea

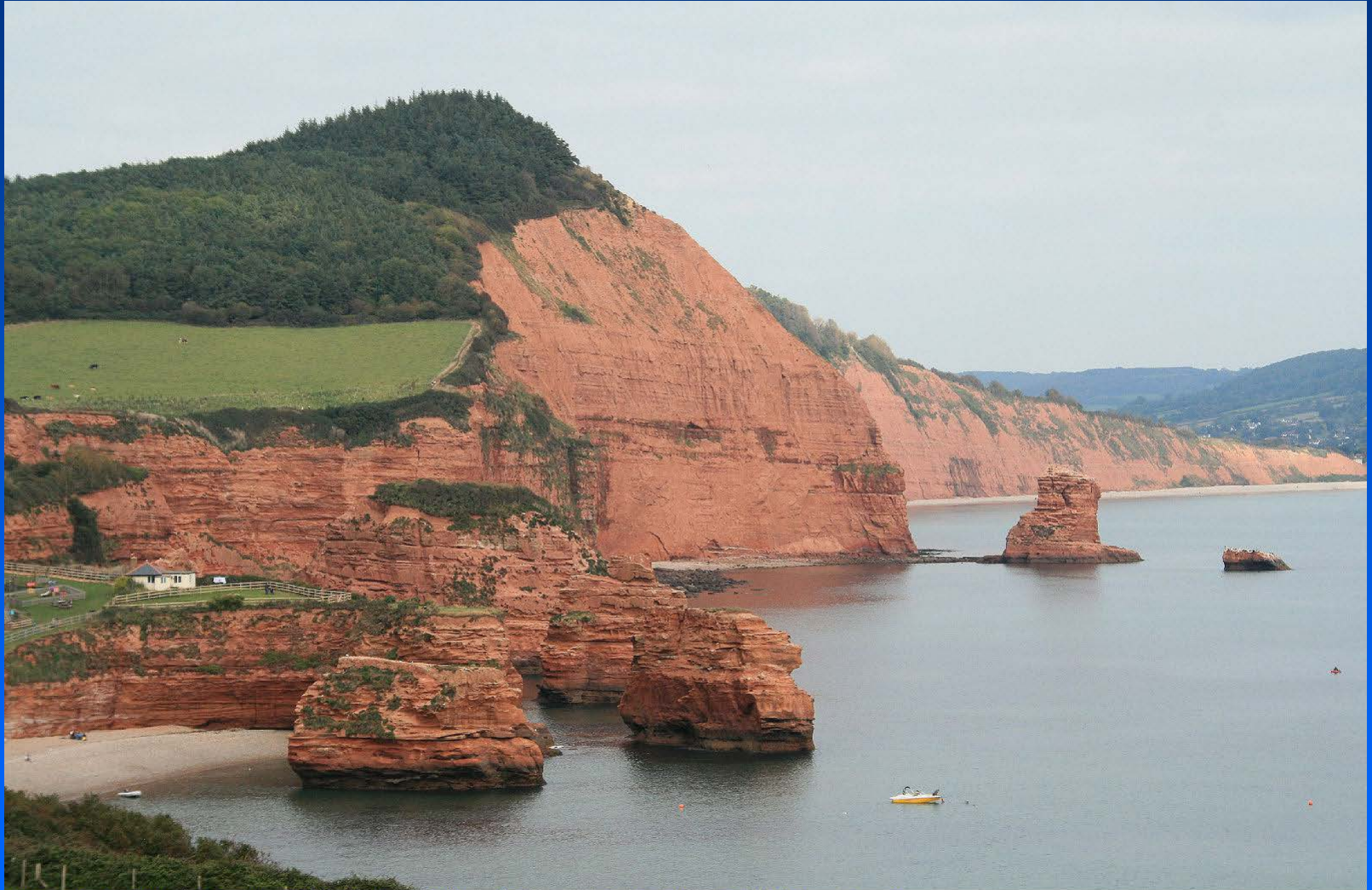




# The Wytch Farm Reservoir



# Wytch Farm Geology



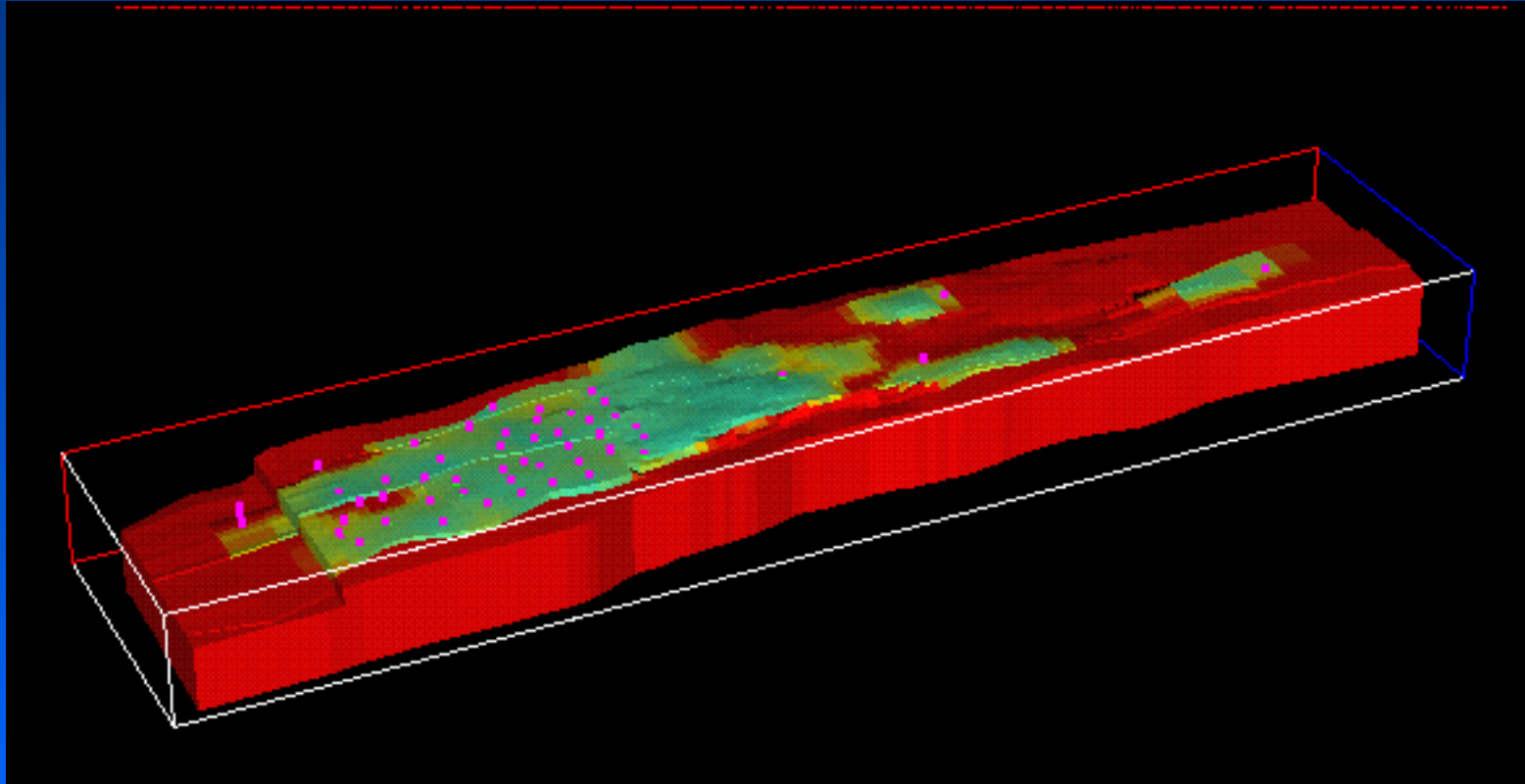
Cliffs of Triassic red beds at Ladram Bay, High Peak and towards Sidmouth, Devon, seen from Smallstones Point, looking northeast. The sequence is of the Otter Sandstone Formation (Ladram Bay cliffs and the various stacks), part of the Sherwood Sandstone Group, and the finer-grained Mercia Mudstone (upper part of the High Peak cliff). Sidmouth is at the right margin of the photograph. Photo: 26th September 2009, about 3 pm, ebbing tide. Ian West (c) 2009.

# Close up of Sherwood Sandstone Outcrop



Cross-stratified Otter Sandstone Formation, Sherwood Sandstone Group, Trias, in the centre of Ladram Bay, near Sidmouth. The cross stratification is of fluvial origin in a semi-arid to arid environment and is probably, at least in part of flash-flood origin. The scale is a one metre rule.  
Photograph: 26th September 2009. Ian West (c) 2009.

# Wytch Farm Reservoir Model



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# Mathematics of Flow in Porous Media

- Conservation of Mass
- Conservation of Momentum
  - replaced by Darcy's law

$$\mathbf{v} = -\frac{k(\mathbf{x})}{\mu} \nabla p$$

- Conservation of Energy
  - most processes isothermal
- Equation of State

# Equations governing flow

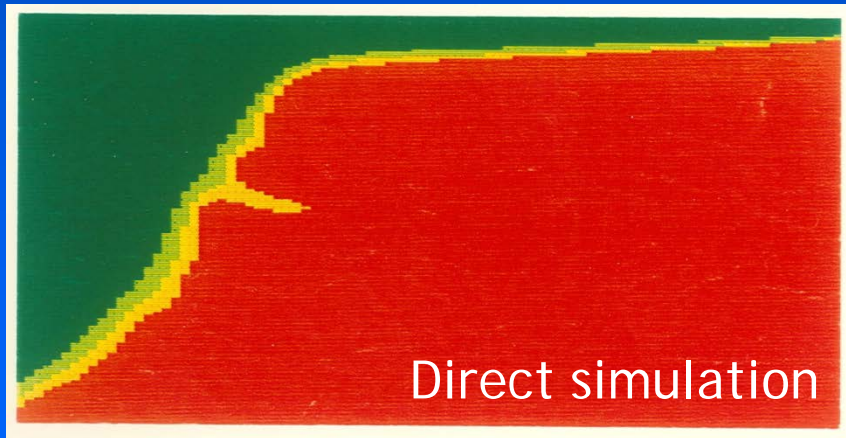
- Parabolic equation for pressure

$$c \frac{\partial p}{\partial t} = \nabla \cdot \left( k(\mathbf{x}) \left( \frac{k_{ro}(S)}{\mu_o} + \frac{k_{rw}(S)}{\mu_w} \right) \nabla p \right)$$

- Hyperbolic equation for saturation

$$\phi(\mathbf{x}) \frac{\partial (\rho_o x_i S_o + \rho_g y_i S_g)}{\partial t} + \nabla \cdot (\rho_o x_i \mathbf{v}_o + \rho_g y_i \mathbf{v}_g) = 0$$

# Low Rate Bead Pack Experiment





# CT Scanned Rock Slab Experiment

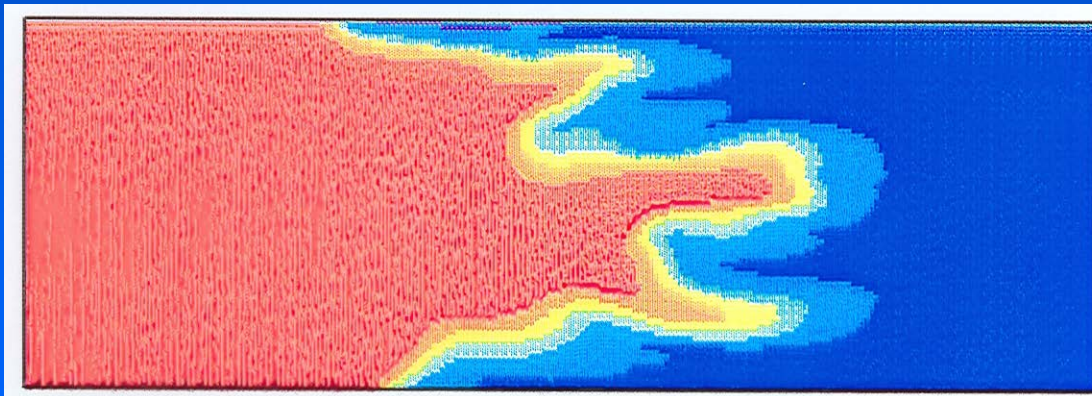
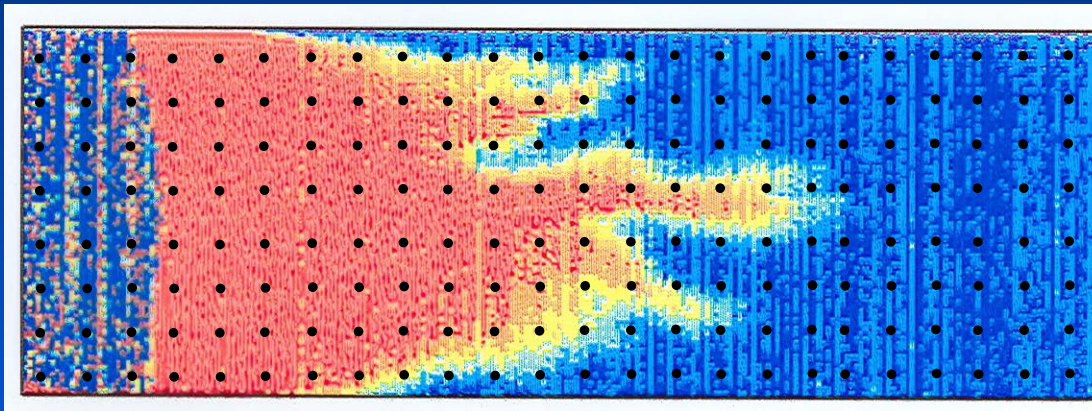


Image from Davies, Muggerridge, Jones, "Miscible Displacements in a Heterogeneous Rock: Detailed Measurements and Accurate Predictive Simulation" SPE 22615 (1991)

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# Uncertainty in Reservoir Description



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# Equations governing flow

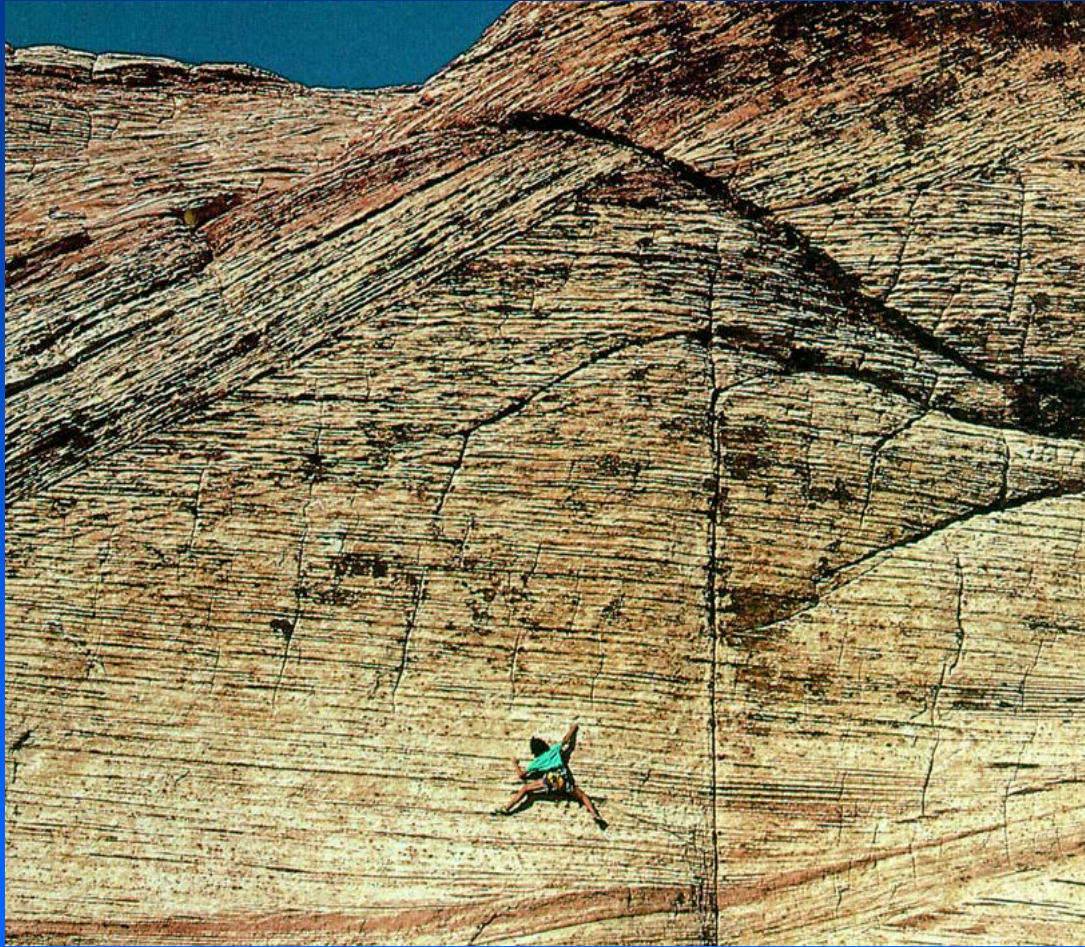
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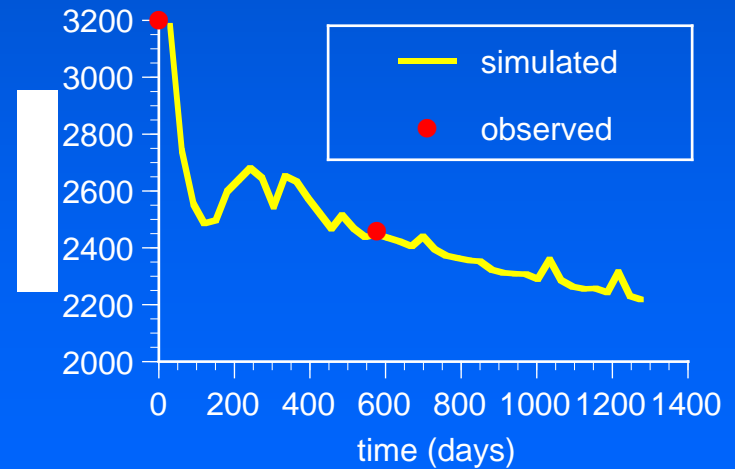
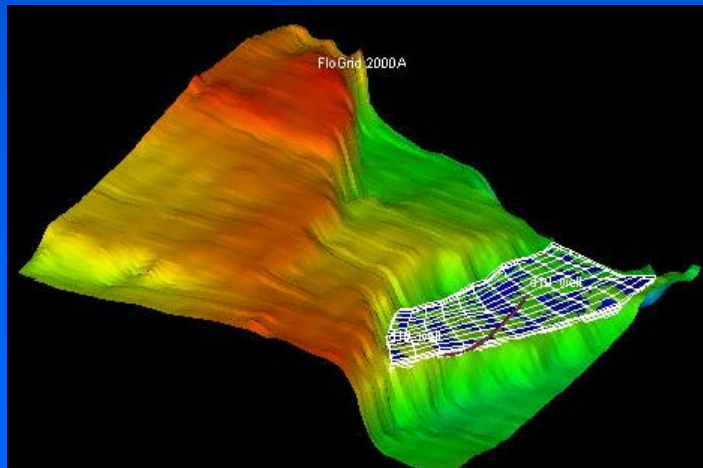
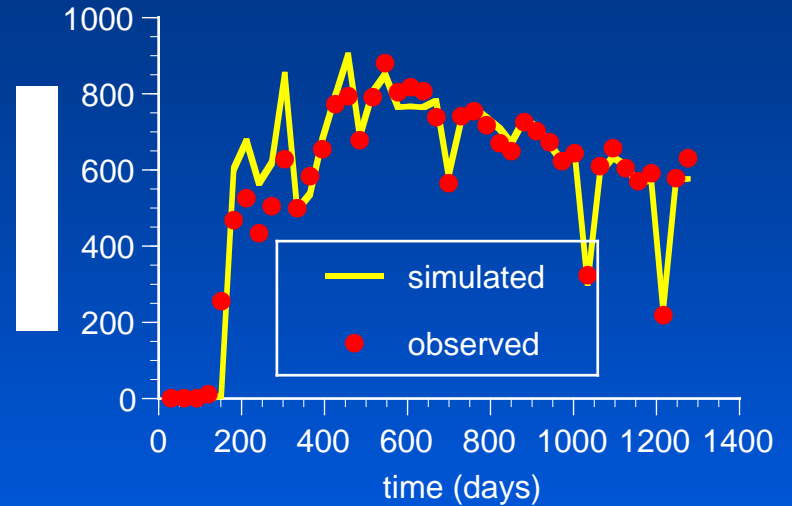
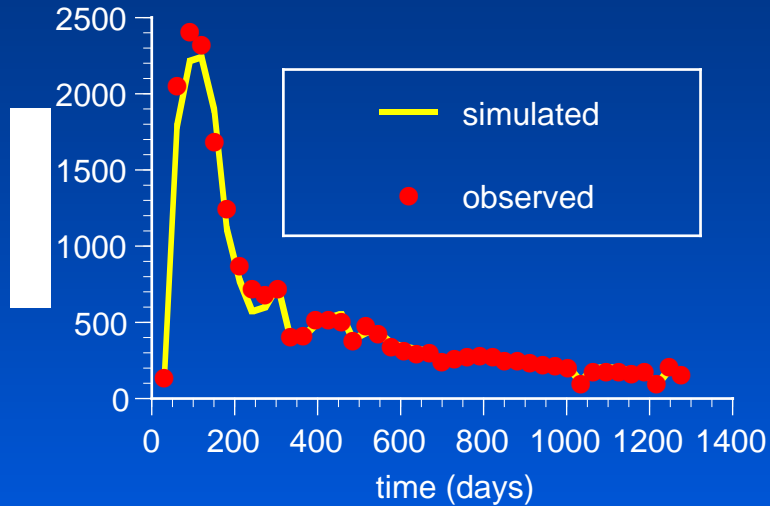
# Data Collection



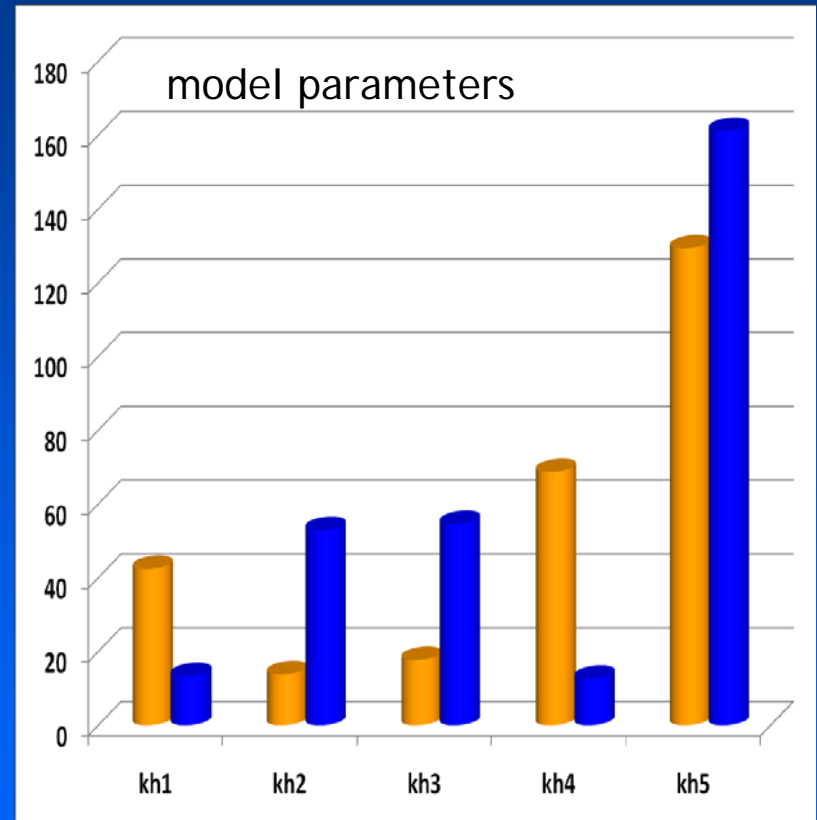
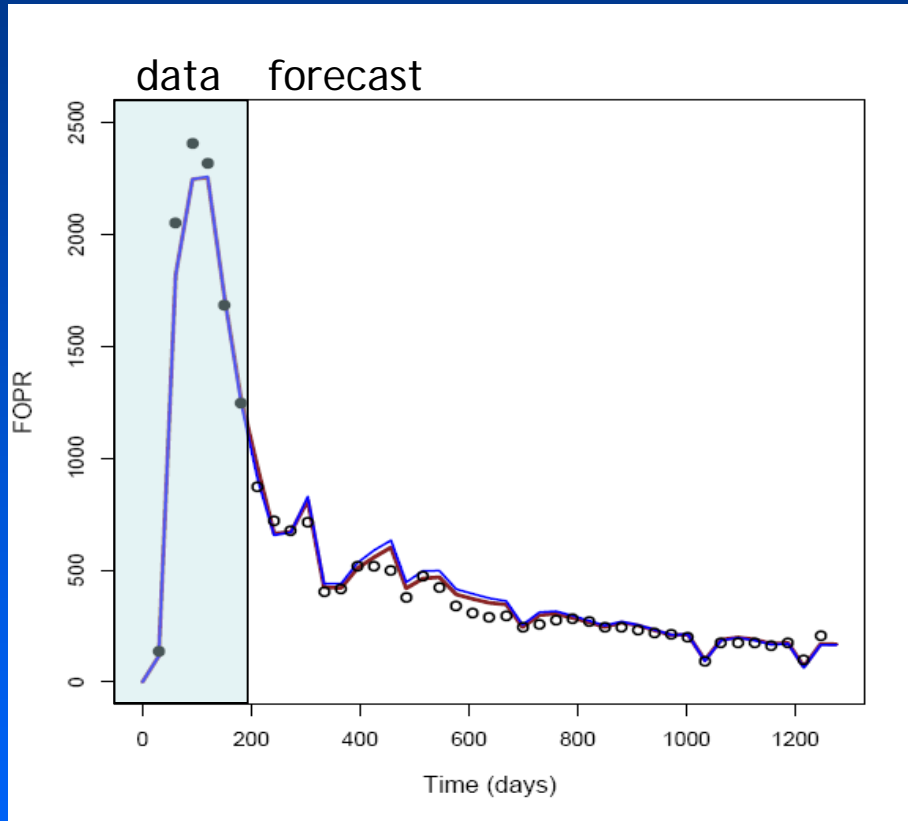
$$\phi(\mathbf{x}) = ?$$

$$k(\mathbf{x}) = ?$$

# Model Calibration: Teal South

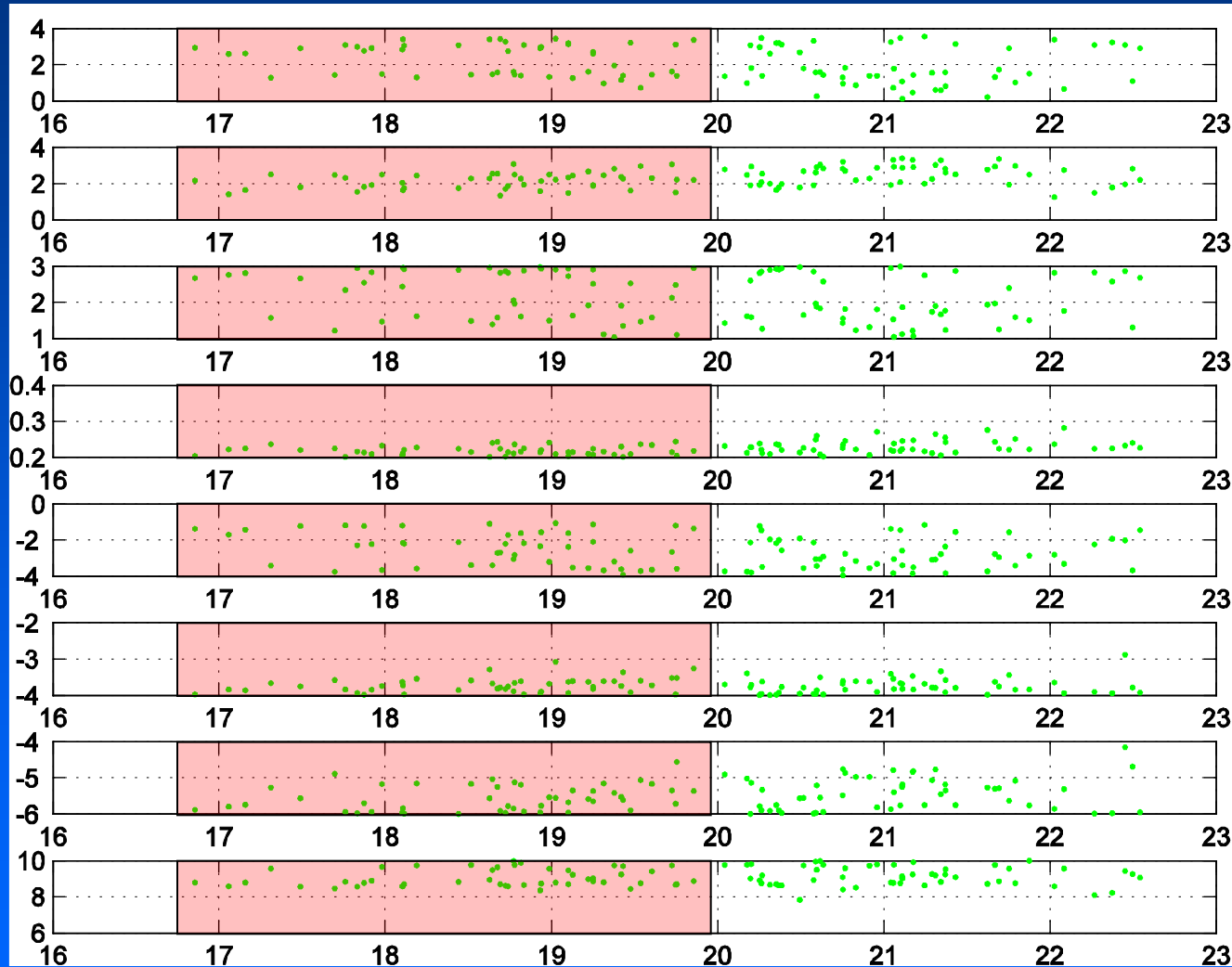


# Calibrated Models are Non-Unique





# Range of Possible Values for Unknown Parameters



$\log(kh_1)$

$\log(kh_2)$

boundary

porosity

$\log(kv/kh_1)$

$\log(kv/kh_1)$

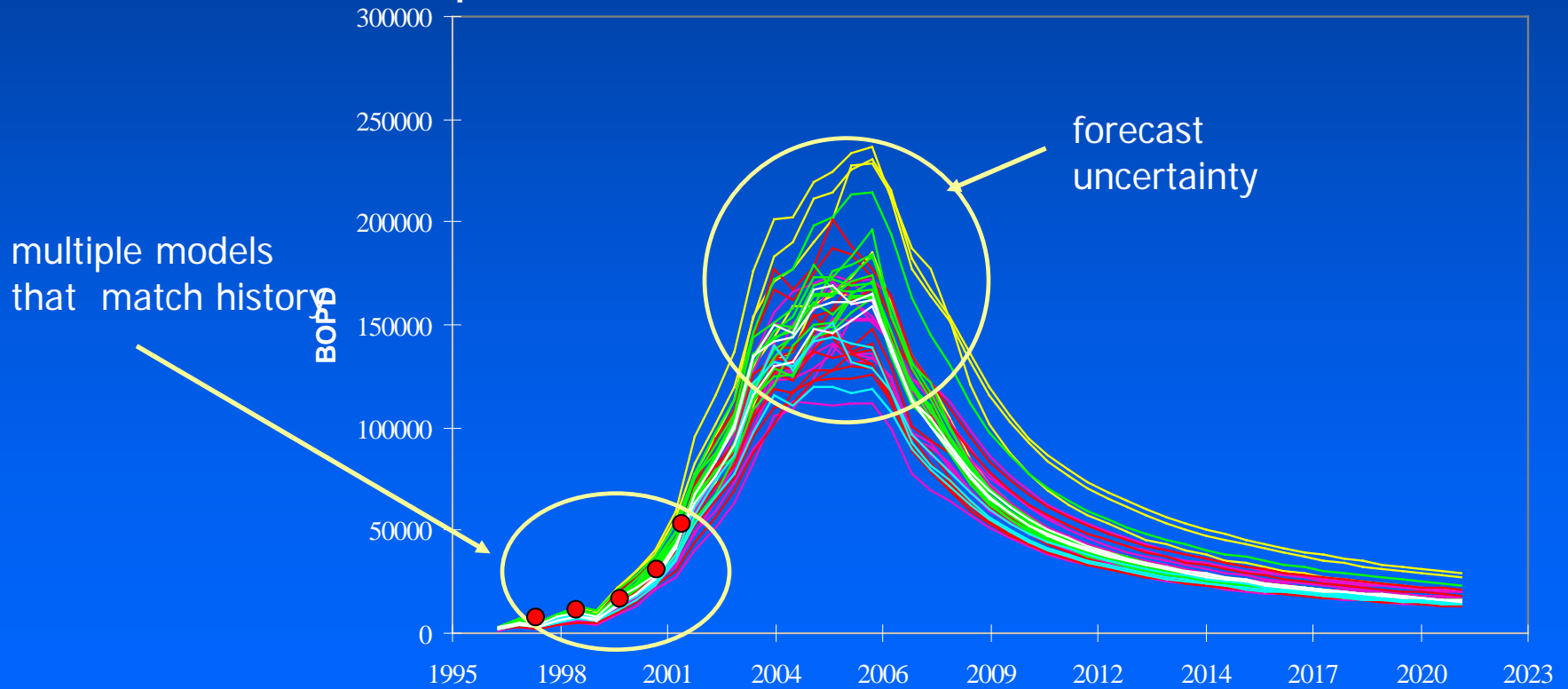
$\log(c_r)$

aquifer  
strength

# Uncertainty Quantification

Generate multiple models that agree with known data

Use models to predict future behaviour



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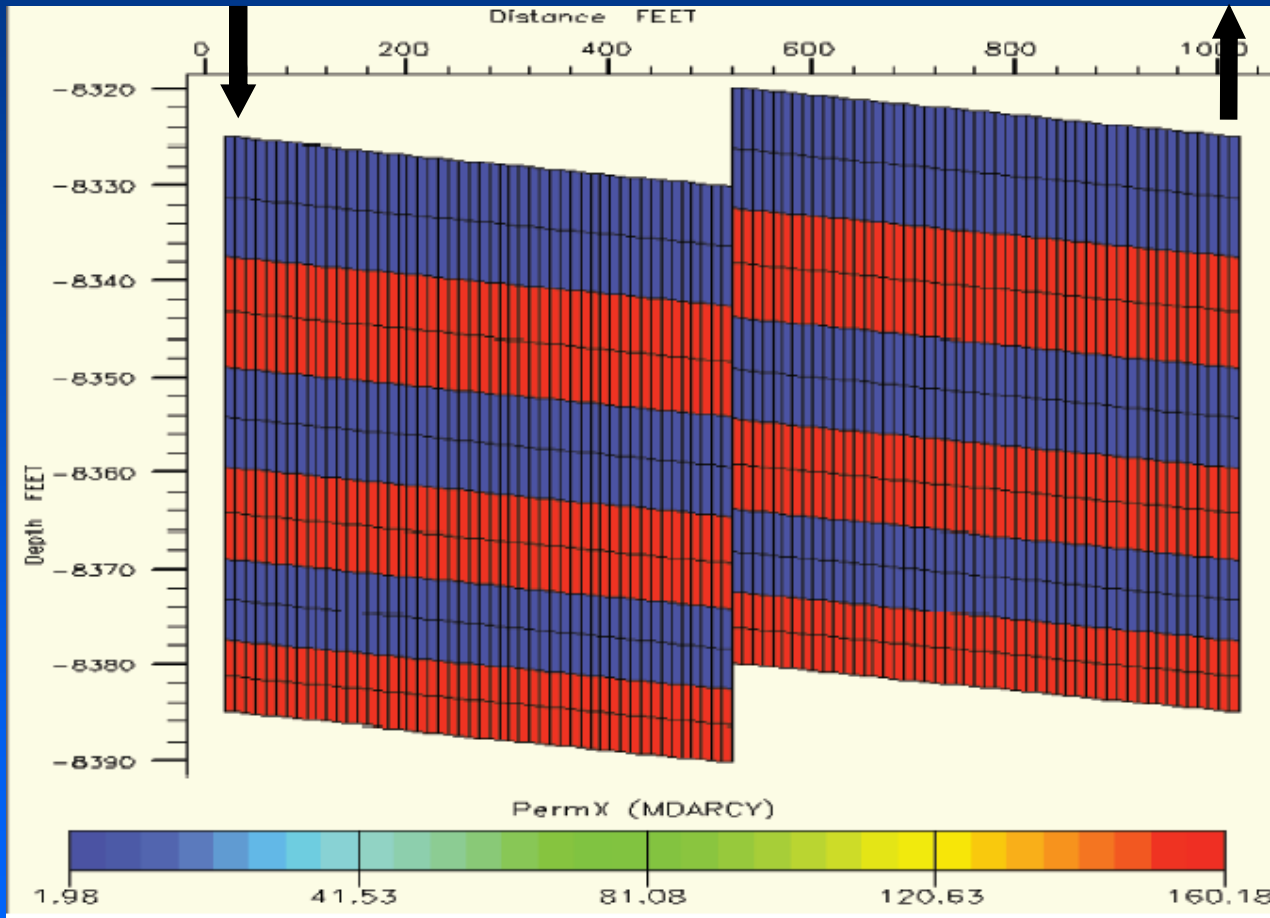
# What kind of problems?

- History matching
  - Generate multiple models consistent with data
- Forecasting
  - Predicting likely production (including range)
- Optimisation
  - Decision making, under uncertainty

# IC Fault Model

Injector

Producer



3 Parameters

$$k_{\text{low}} = [0 \text{ .. } 50] \text{ md}$$

$$k_{\text{high}} = [100 \text{ .. } 200] \text{ md}$$

$$\text{throw} = [0 \text{ .. } 60] \text{ ft}$$

Objectives (rates)

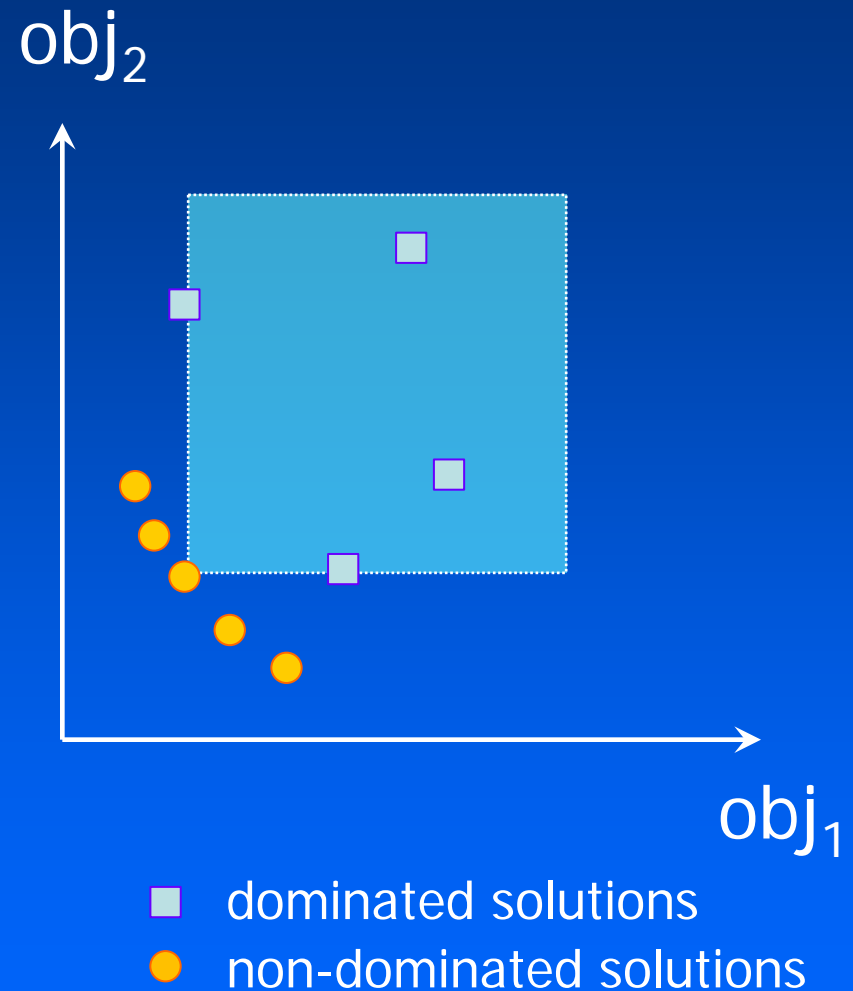
$$\text{obj1} = \text{FOPR}$$

$$\text{obj2} = \text{FWPR}$$

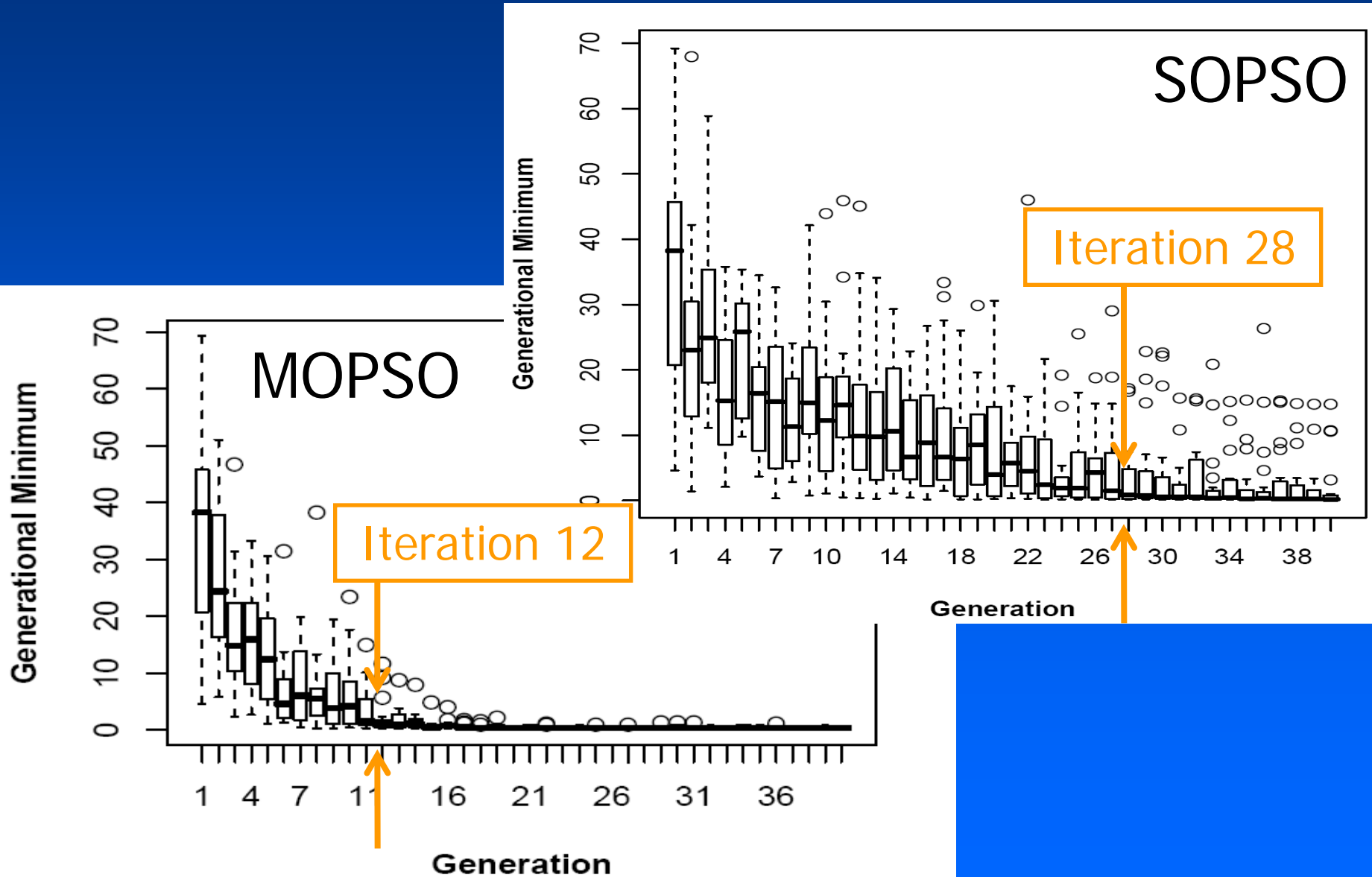
$$\text{Misfit} = \text{obj1} + \text{obj2}$$

# Multi-Objective PSO: Pareto Dominance

- 1) A is not worse than B in all objectives
- 2) A is strictly better than B in at least one objective

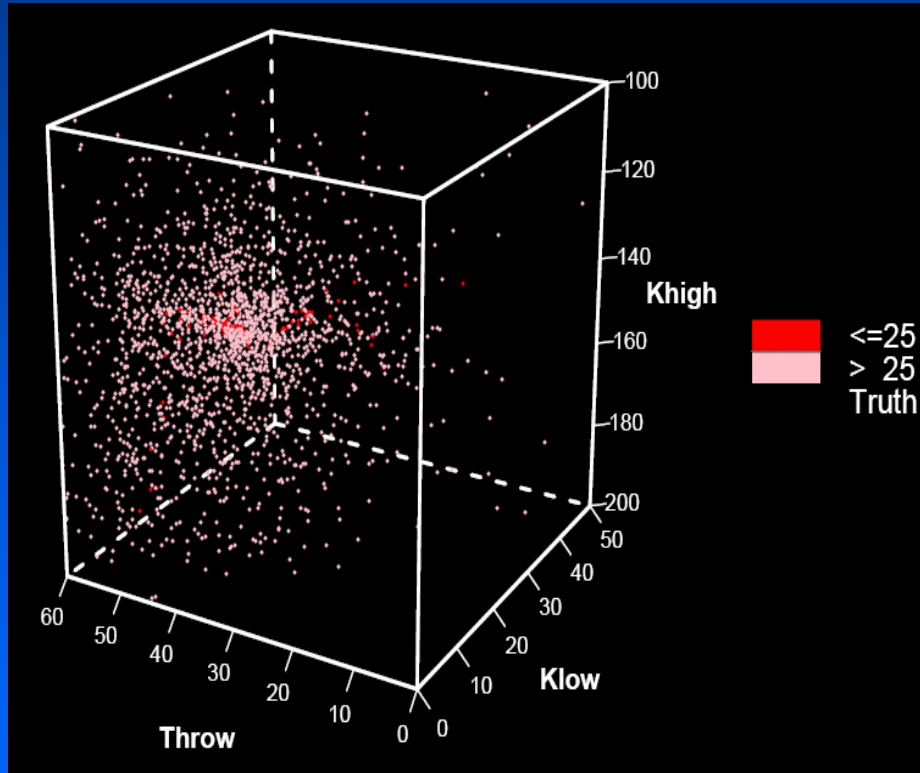


# Convergence Speed – 20 Runs

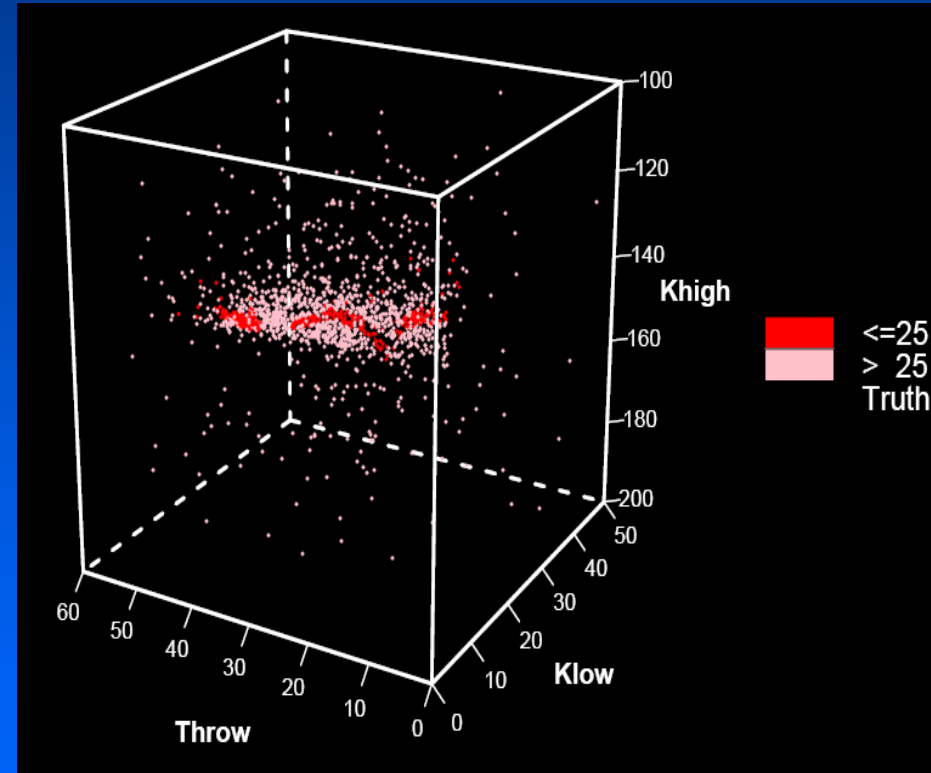


# Diversity of Models

## SOPSO



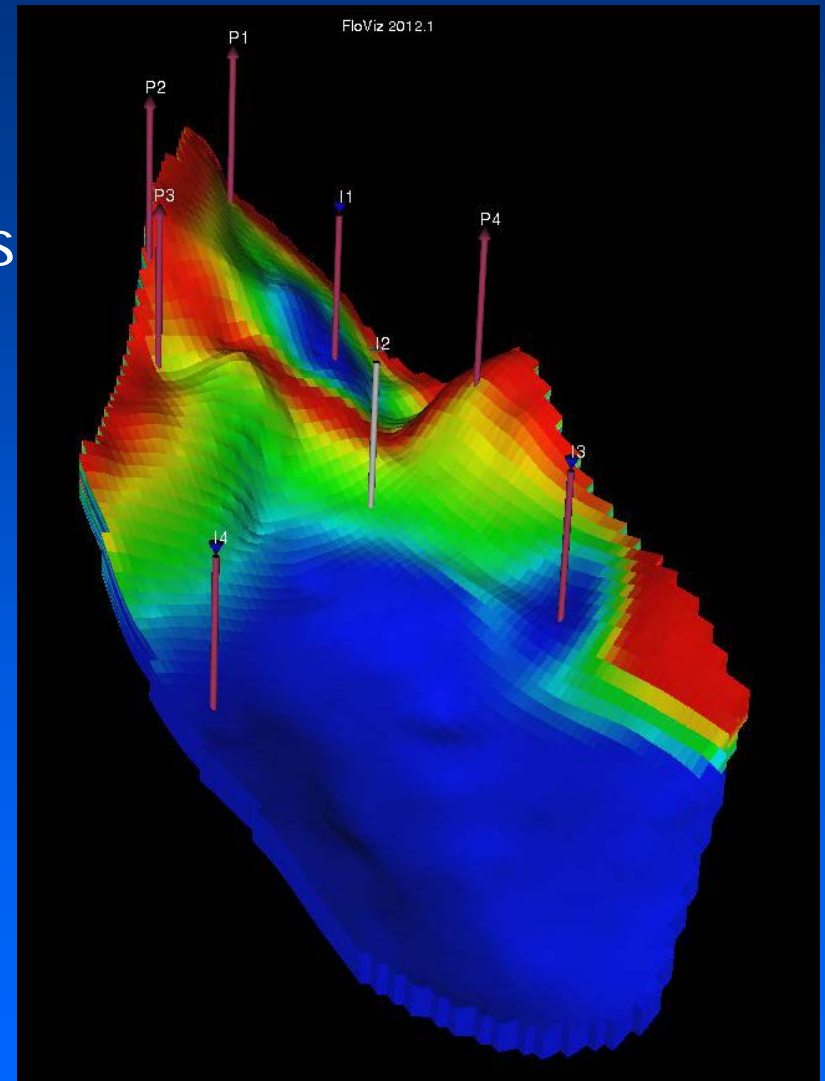
## MOPSO





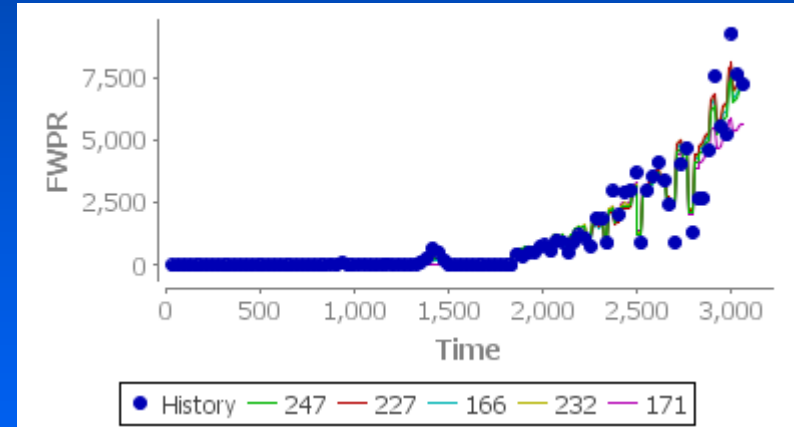
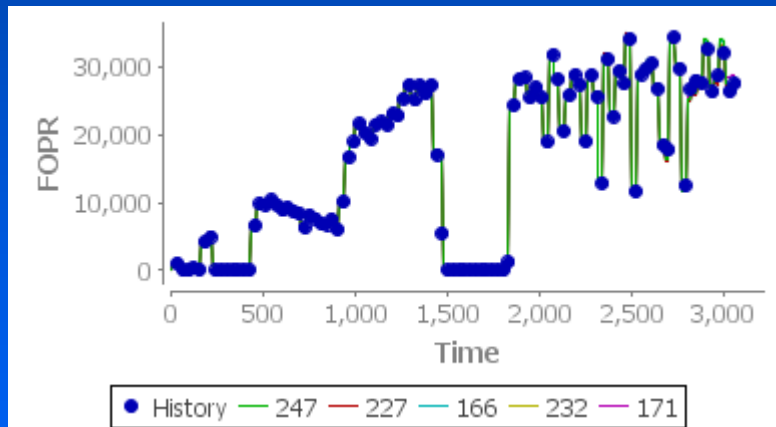
# Optimisation

- Scapa field
  - Used for student projects



# Field Level History Match

- Match to first 10 years of history
  - Remaining data used to check forecast

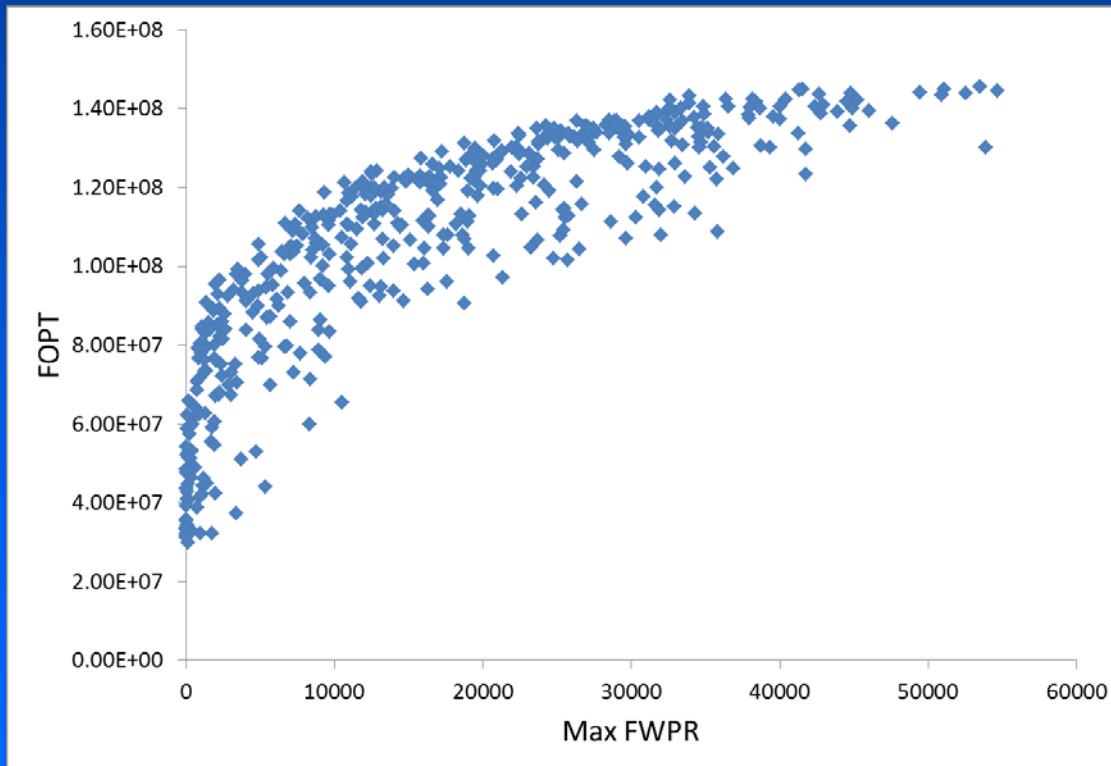


# Optimisation

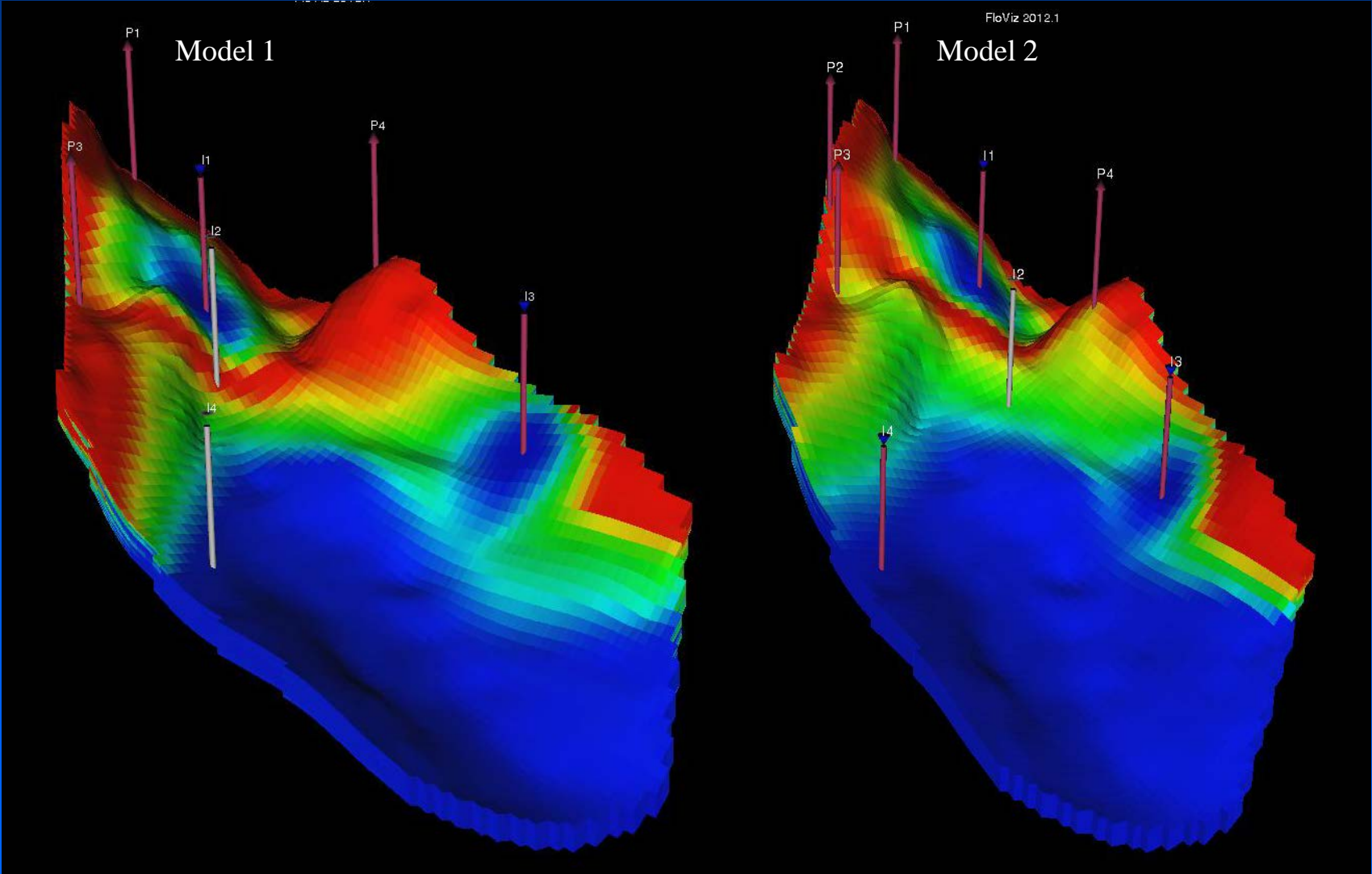
- Multi-Objective
  - Maximise cumulative oil (FOPT)
  - Minimise maximum water rate (FWPR)
- Optimisation variables
  - Well locations (16 variables –  $i, j$  for each well)
  - Injection well rates
  - Injection well status ('open' vs 'shut')

# Optimise Well Locations and Injection Rates

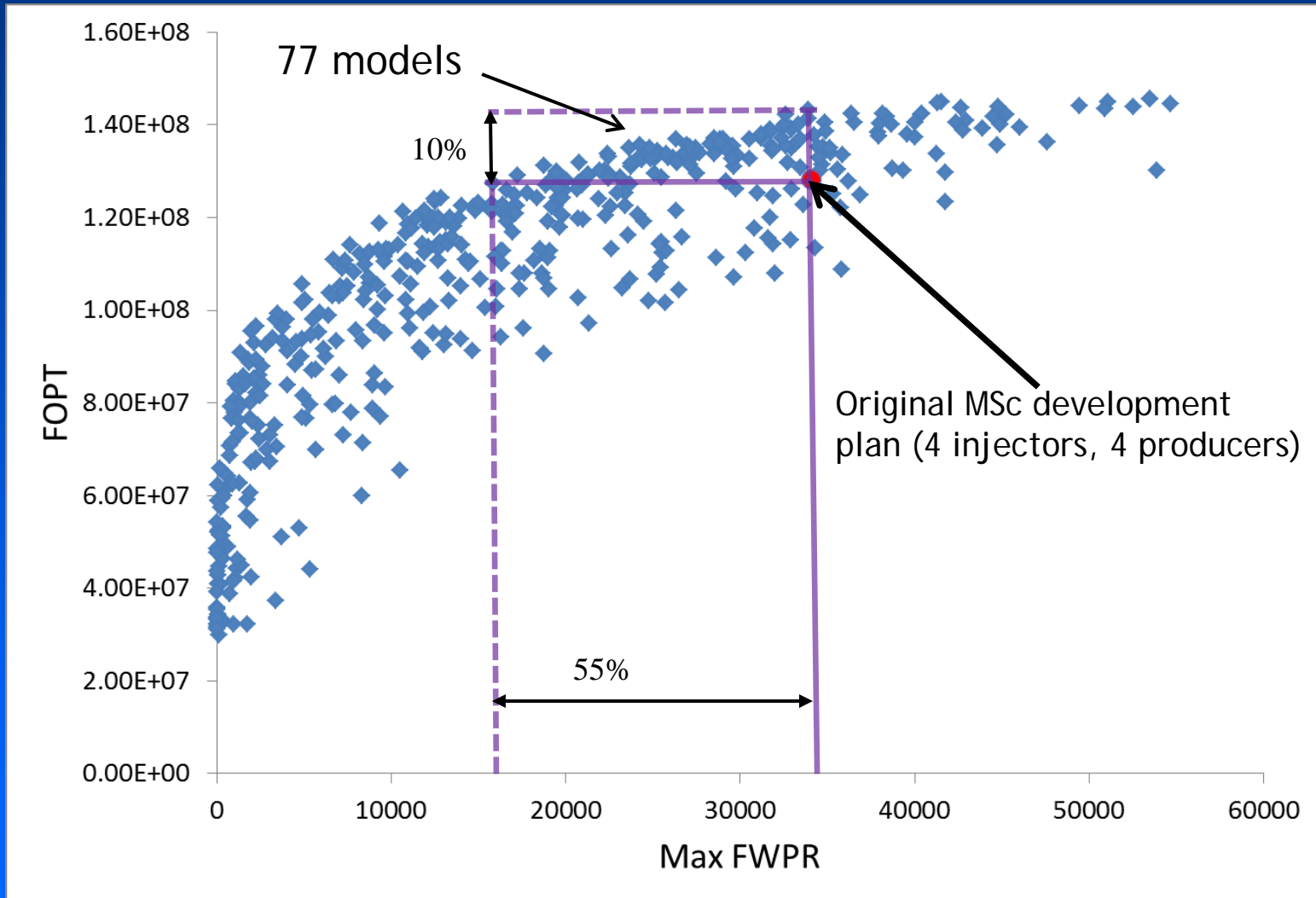
- Vary injection well rates and locations of wells
  - Well rates in  $[0,15]$  MBD



# Oil Saturation Comparison



# Comparison with Original MSc Plan



# Where Does EQUIP come in?

- 3 factors
  - Data input is uncertain
  - Models take time to run
  - Decisions are costly
- Need
  - Fast, effective model calibration techniques
  - Combine maths, stats, computer science, engineering