Computation in Mathematics and Mathematics in Computation

March 2024

Andrew Burbanks - *Computer-assisted proofs for renormalisation fixed-points*

Abstract: We will show some examples of using computer-assisted techniques (validated numerics in function spaces) to prove the existence of fixed points of renormalisation operators and to help explore their properties. One of the simplest examples is for period doubling in the dynamics of families of maps of the interval with even degree critical point. We use rigorous computer-assisted techniques to bound operations in a space of analytic functions and hence show that a Newton-like operator for the fixed-point problem is a contraction map on a suitable ball. We bound the spectrum of the Frechet derivative of the renormalisation operator at the fixed point, establishing the hyperbolic structure, in which the presence of a single essential expanding eigenvalue helps to explain common features observed in families of maps lying in the relevant universality class. Our computations use multi-precision interval arithmetic with rigorous directed rounding modes to bound tightly the coefficients of the relevant power series and their high-order terms, and the corresponding universal constants.

Igors Gorbovickis - *Critical points of the multipliers*

Abstract: A parameter $c_0 \in \mathbb{C}$ in the family of quadratic polynomials $f_c(z) = z^2 + c$ is a *critical point of a period n multiplier*, if the map f_{c_0} has a periodic orbit of period n, whose multiplier, viewed as a locally analytic function of c, has a vanishing derivative at $c = c_0$. Information about the location of the critical points and critical values of the multipliers can be helpful for understanding the geometry of the Mandelbrot set. In this talk we will discuss some results about the critical points of the multipliers in the quadratic family. In particular, we will emphasize the interplay between computations and rigorous results, inspired by each other.

Igor Potapov - *Reachability problems in iterative maps and matrix semigroups*

Abstract: A large number of naturally defined matrix problems are still unanswered, despite the long history of matrix theory. In this presentation, I will discuss several challenging computational problems for matrix semigroups and their connections to matrix equations, iterative maps and linear recurrence sequences. Some of these problems are still unsolved but at the same time identified as reference points in the algorithmic analysis of reachability problems. The cornerstones of these reachability problems are in the synergy of the questions in mathematics and computer science: computability, automata theory, matrix theory, combinatorics, abstract algebra, number theory, etc. The main objective of future research is to develop new concepts and methods combining symbolic and numerical techniques as well as unifying the results in automata, computability, number and matrix theories.

James Worrell - *On Transcendence of Sturmian and Arnoux-Rauzy Words*

Abstract: We consider numbers of the form $\alpha := \sum_{n=0}^{\infty} \frac{u_n}{\beta^n}$, where (u_n) is an infinite word over a finite alphabet and β is a complex number of absolute value greater than one. We present a combinatorial criterion on u, called echoing, that implies that α is transcendental whenever β is algebraic. We show that every Sturmian word is echoing, as is the Tribonacci word, a leading example of an Arnoux-Rauzy word. We give an application of our transcendence results to the theory of dynamical systems, showing that for a contracted rotation on the unit circle with algebraic slope, its limit set is either finite or consists exclusively of transcendental elements other than its endpoints 0 and 1. This confirms a conjecture of Bugeaud, Kim, Laurent, and Nogueira.