



Learning a Social Network by Influencing Opinions

Dmitry Chistikov, Luisa Estrada,
Mike Paterson and Paolo Turrini

INFLUENCING OPINION DYNAMICS

Suppose you are a campaigner who wants to promote a new ...

law product technology initiative

on a social network where each member has an initial opinion about it (○, ●)

- ✓ You can persuade some members to support your campaign.
- ✓ You know how "people talk" across the social network.



Want to target the members with the strongest influence.

INFLUENCING OPINION DYNAMICS

Suppose you are a campaigner who wants to promote a new ...

law product technology initiative

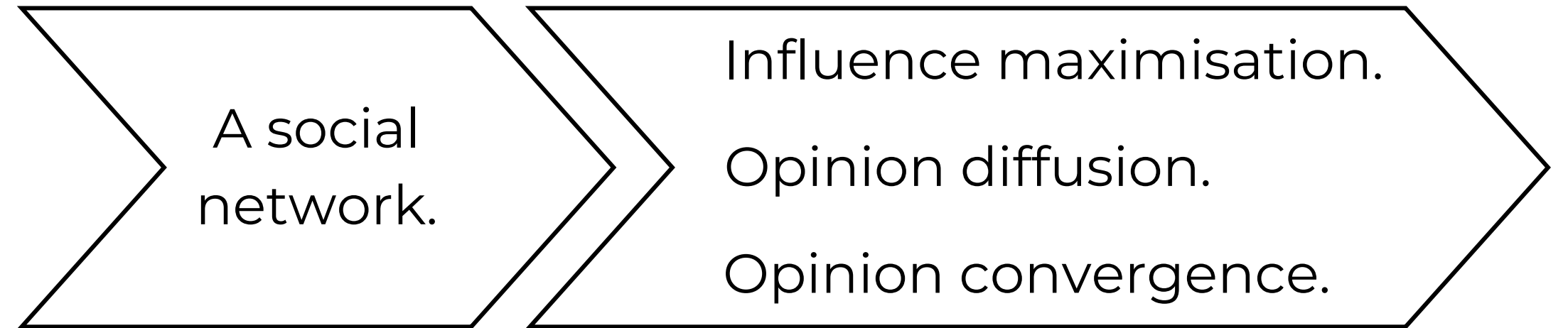
on a social network where each member has an initial opinion about it (○, ●)

- ✓ You can persuade some members to support your campaign.
- ✓ You know **how** "people talk" across the social network.
 - ≠ Knowing **who talks to whom**.

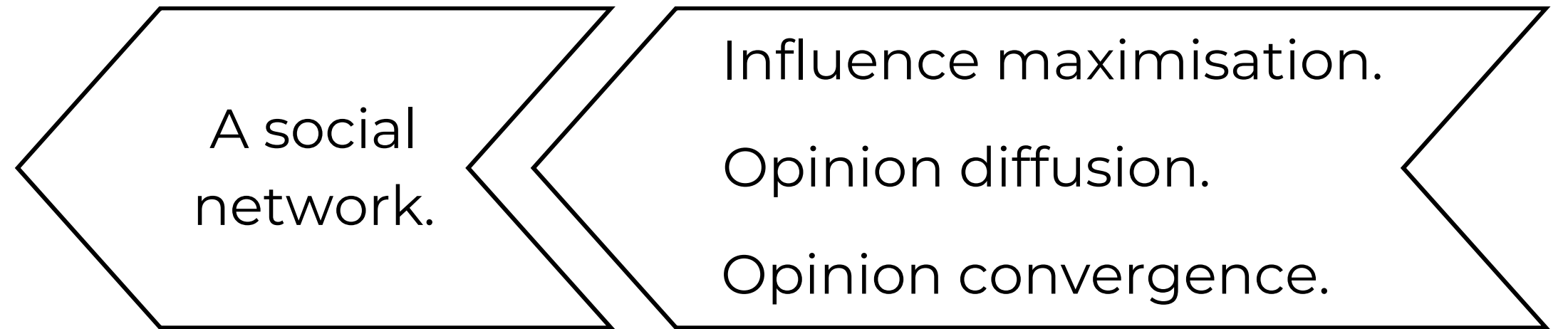


Want to target the members with the strongest influence.

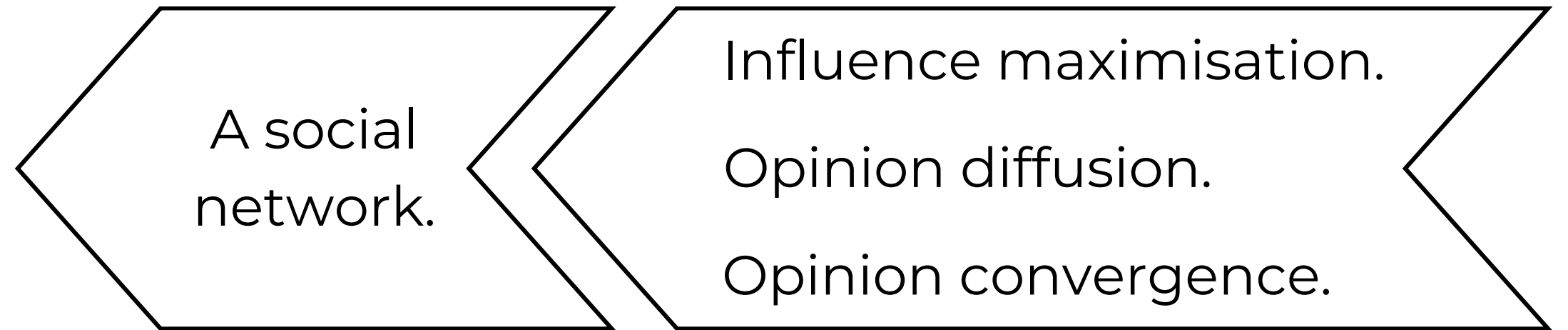
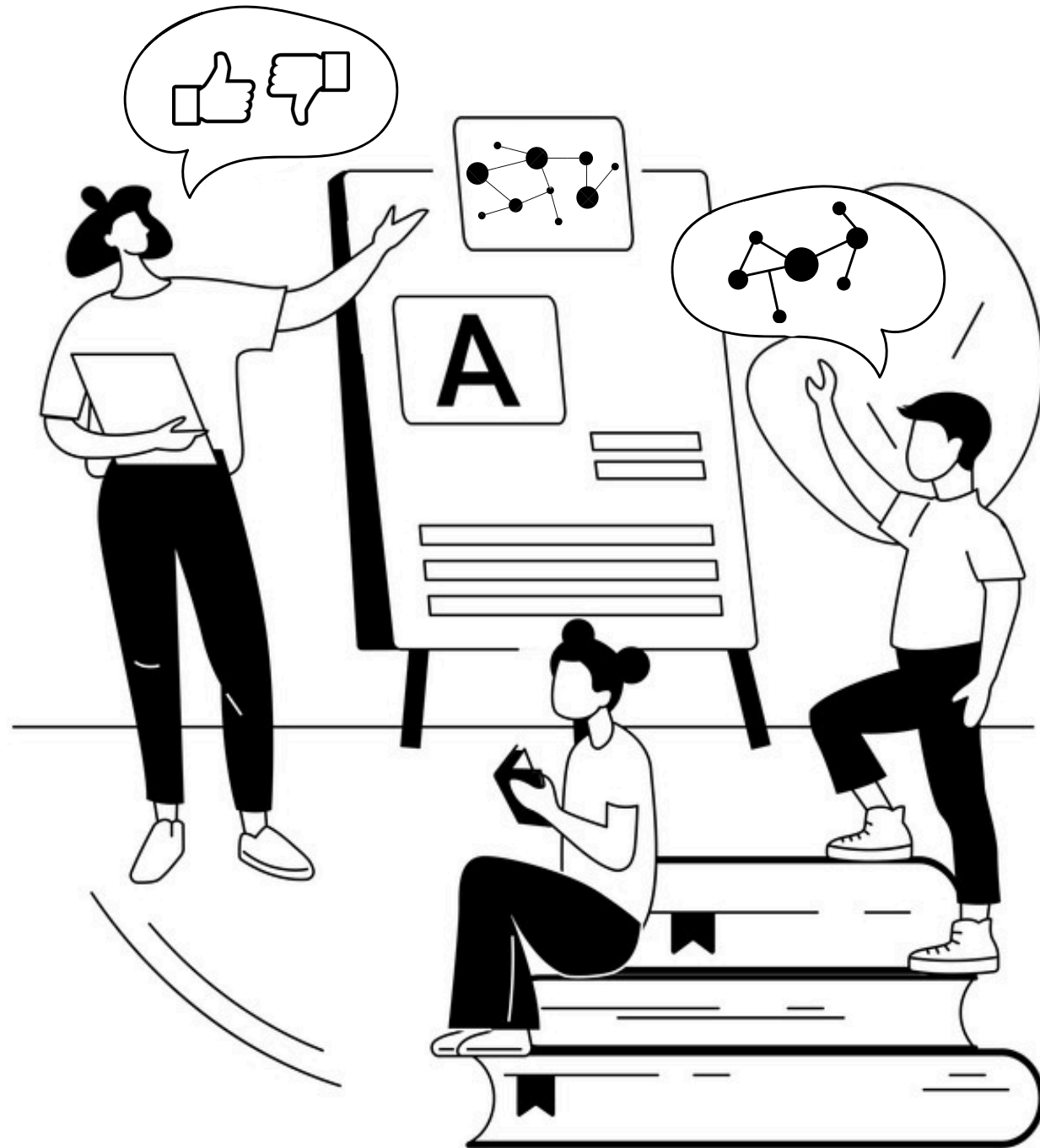
PROBLEM SETUP



PROBLEM SETUP



PROBLEM SETUP



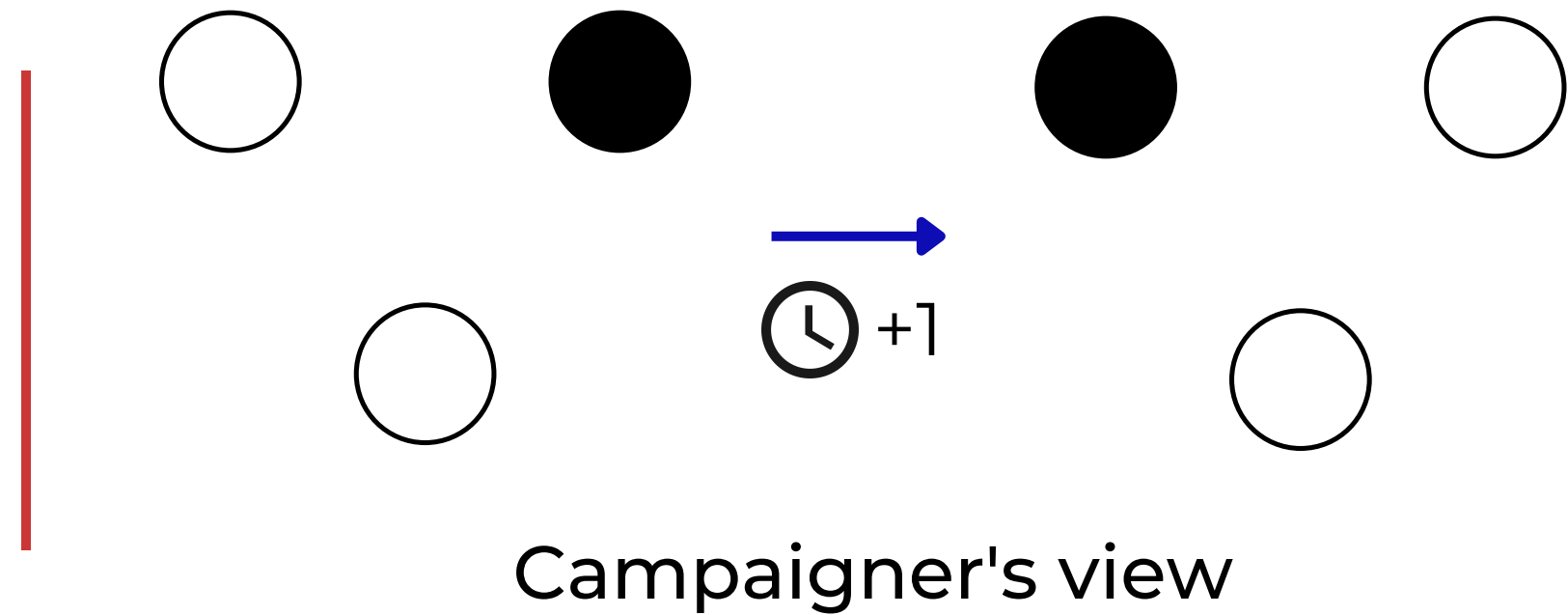
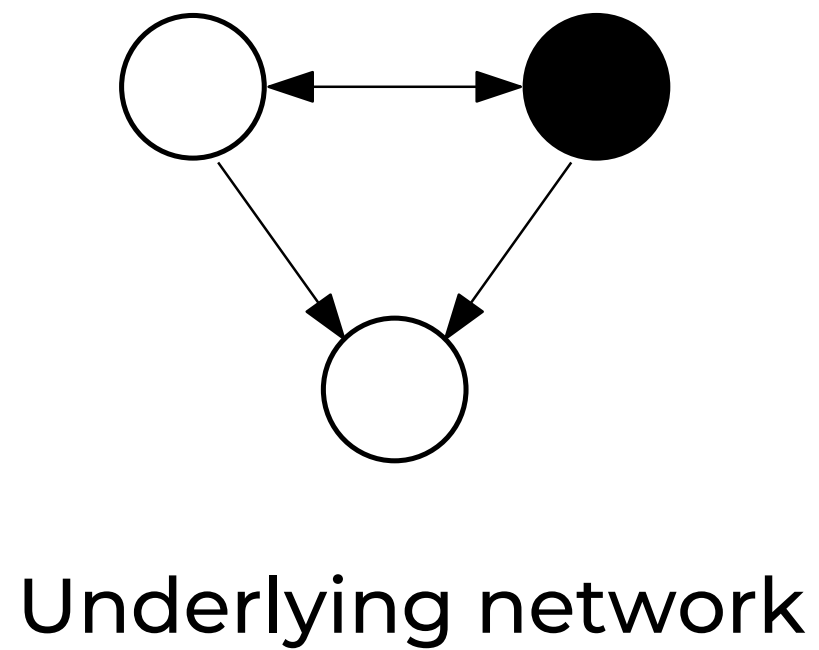
Game rules

- ✗ Connections between agents are hidden.
- ✓ Allow the campaigner to intervene on the agents' opinions and observe the result.
- ✓ Stop once there is only one feasible network.
- ✓ Opinions diffuse following **majority dynamics**.

PROBLEM SETUP: AN EXAMPLE

Task: To learn the underlying network.

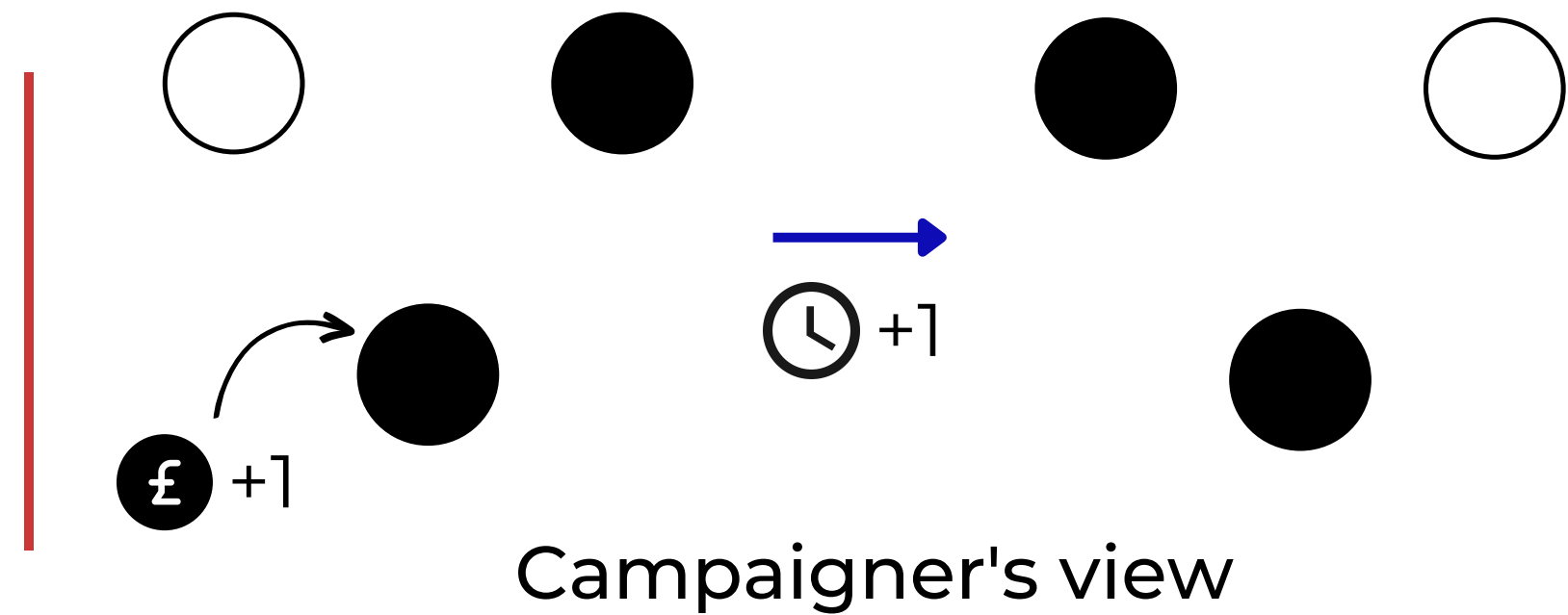
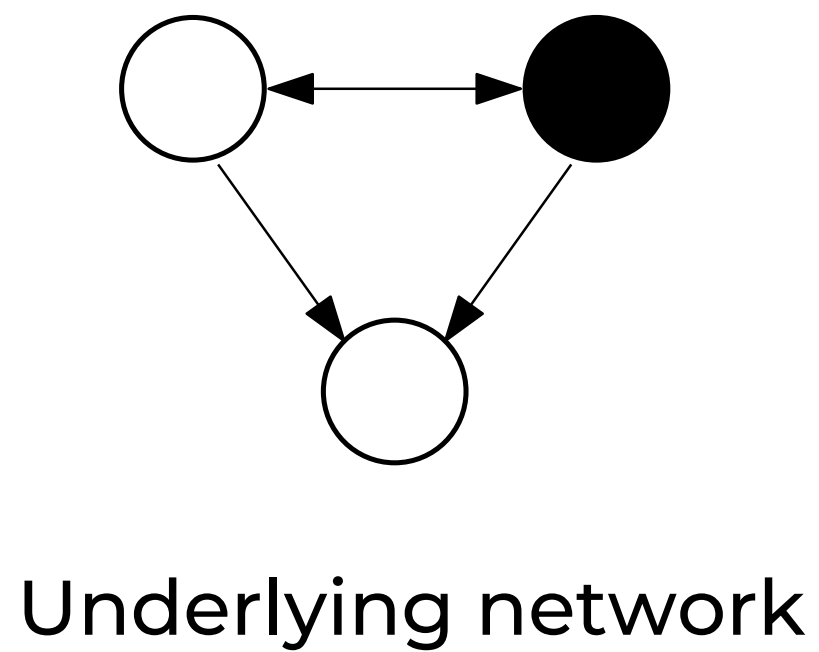
Budget: 🕒 Observations and 💷 interventions



PROBLEM SETUP: AN EXAMPLE

Task: To learn the underlying network.

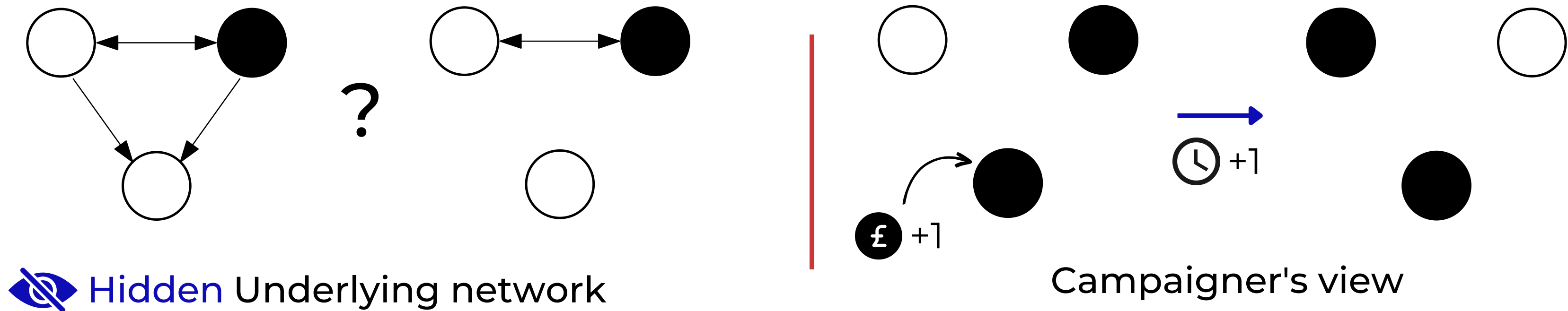
Budget: 🕒 Observations and 💷 interventions



PROBLEM SETUP: AN EXAMPLE

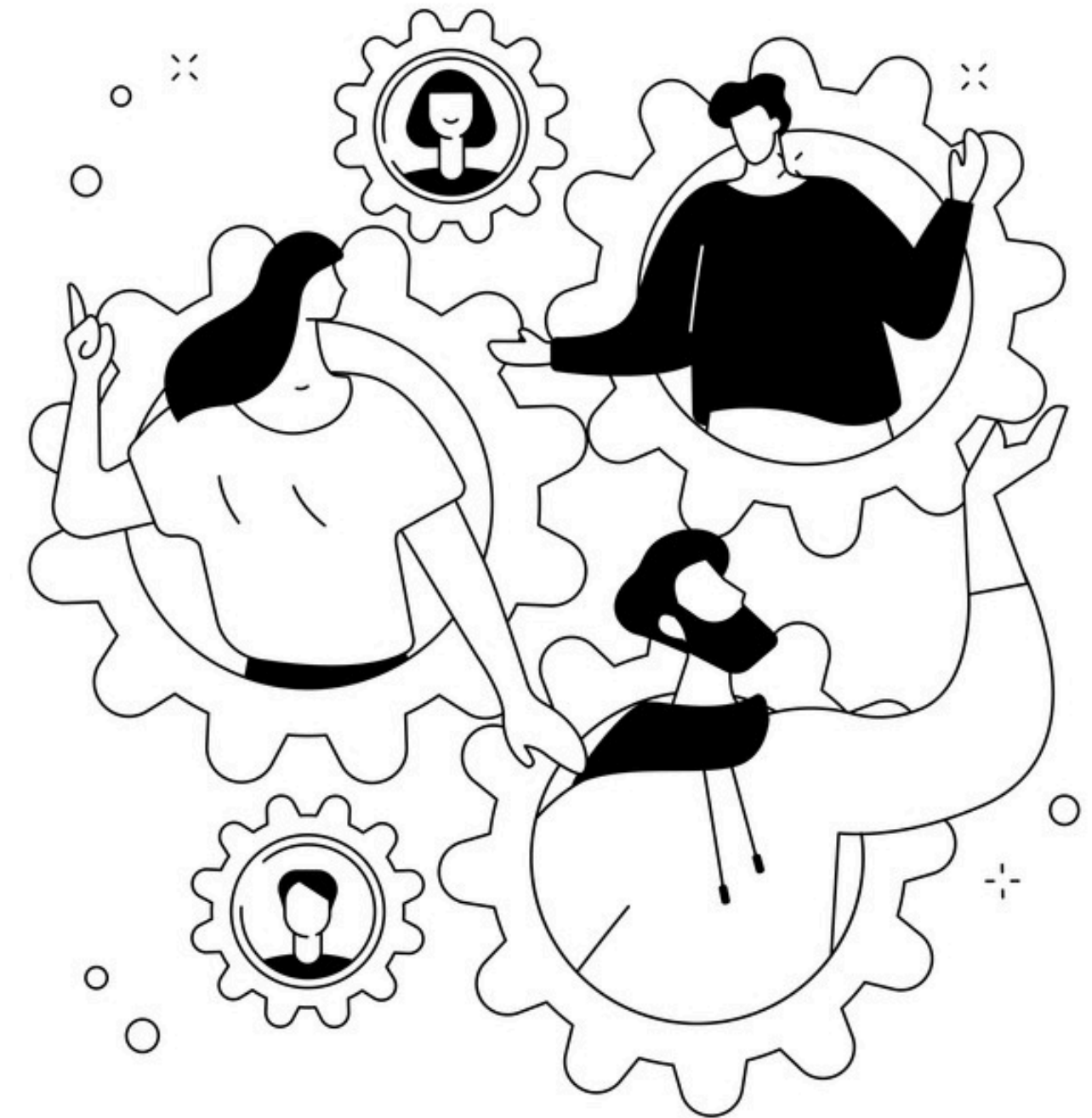
Task: To learn the underlying network.

Budget: 🕒 Observations and 💷 interventions



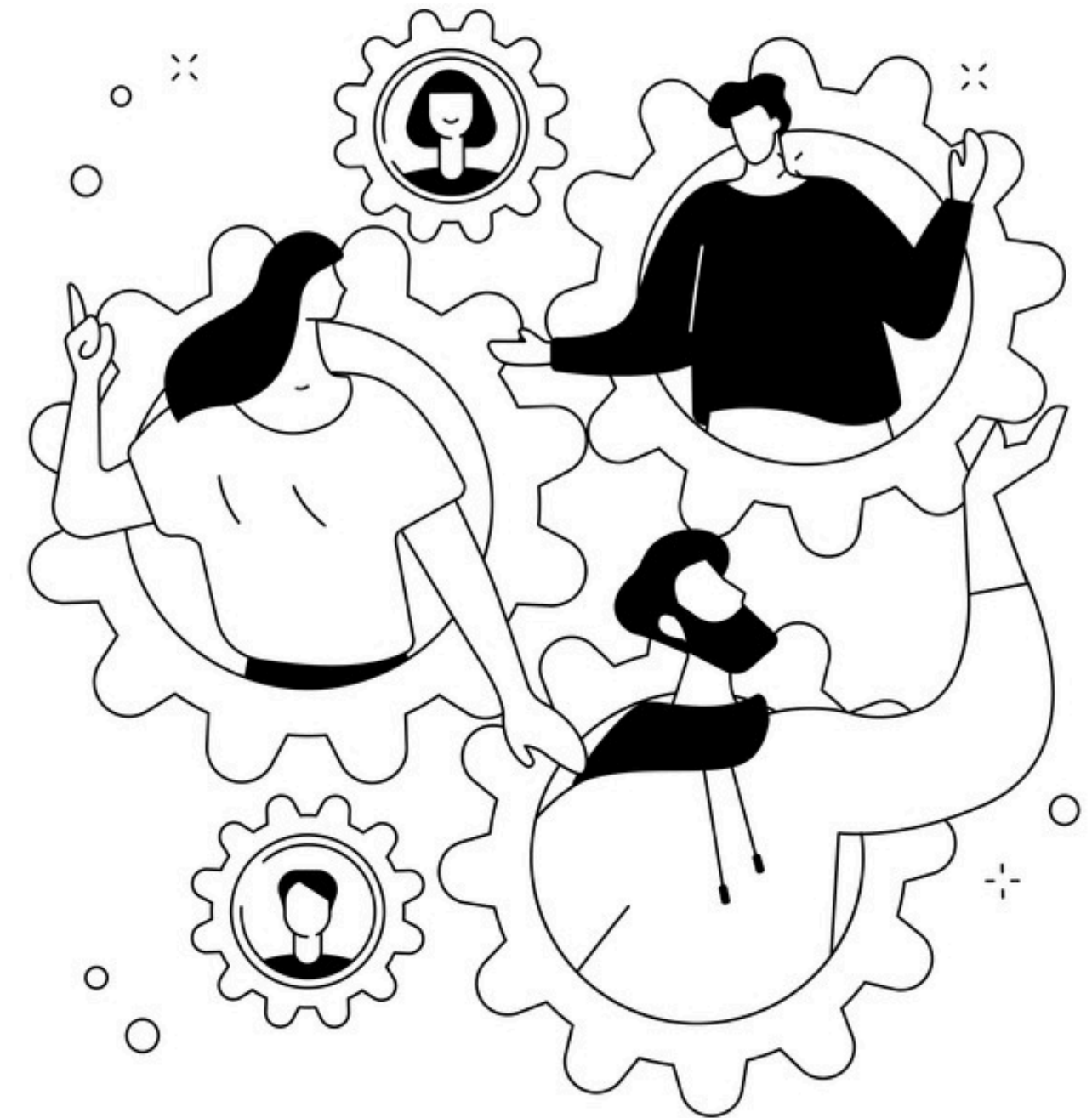
OUR MAIN RESULTS

Learning task	Observation budget	Intervention budget
Identify an even clique	$\mathcal{O}(n)$	$\mathcal{O}(n)$
Identify an odd clique	$\mathcal{O}(n)$	$\mathcal{O}(n^2)$
Learn any network $G \in \mathcal{H}$	$\mathcal{O}(n^2)$	$\mathcal{O}(n^3)$



OUR MAIN RESULTS

Learning task	Observation budget	Intervention budget
Identify an even clique	$\mathcal{O}(n)$	$\mathcal{O}(n)$
Identify an odd clique	$\mathcal{O}(n)$	$\mathcal{O}(n^2)$
Learn any network $G \in \mathcal{H}$	$\mathcal{O}(n^2)$	$\mathcal{O}(n^3)$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

GENERAL CASE

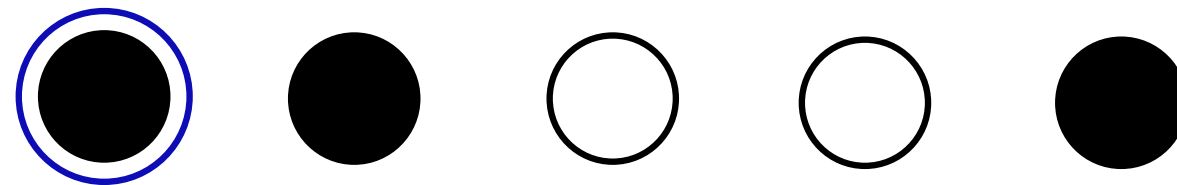
How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

Proof (sketch)

Stage 1: Find the pivot



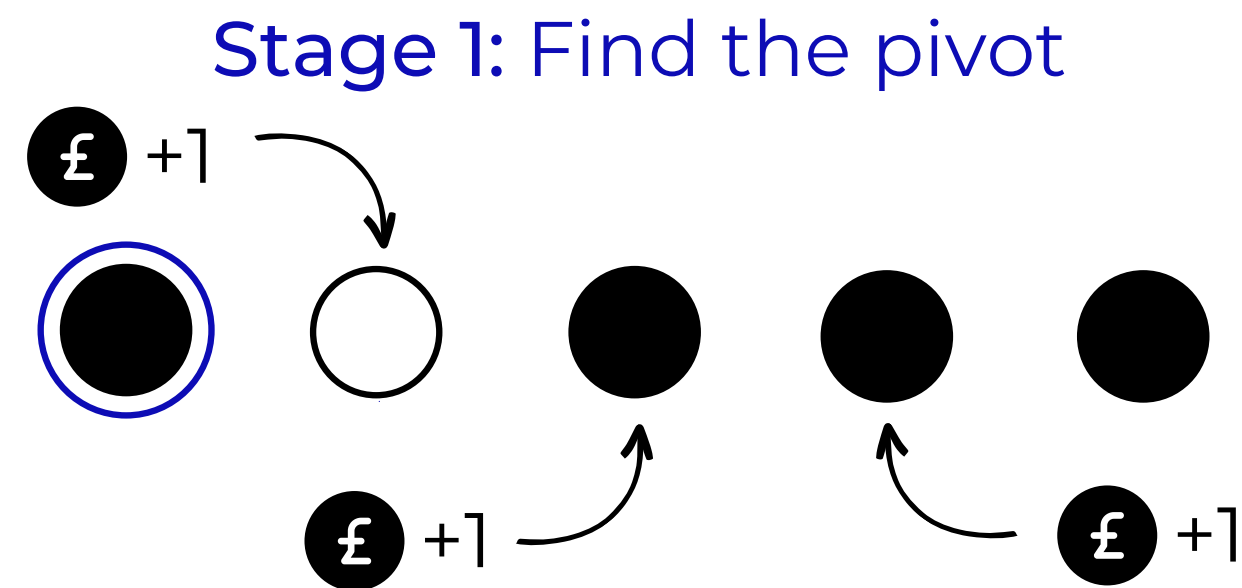
GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

Proof (sketch)



GENERAL CASE

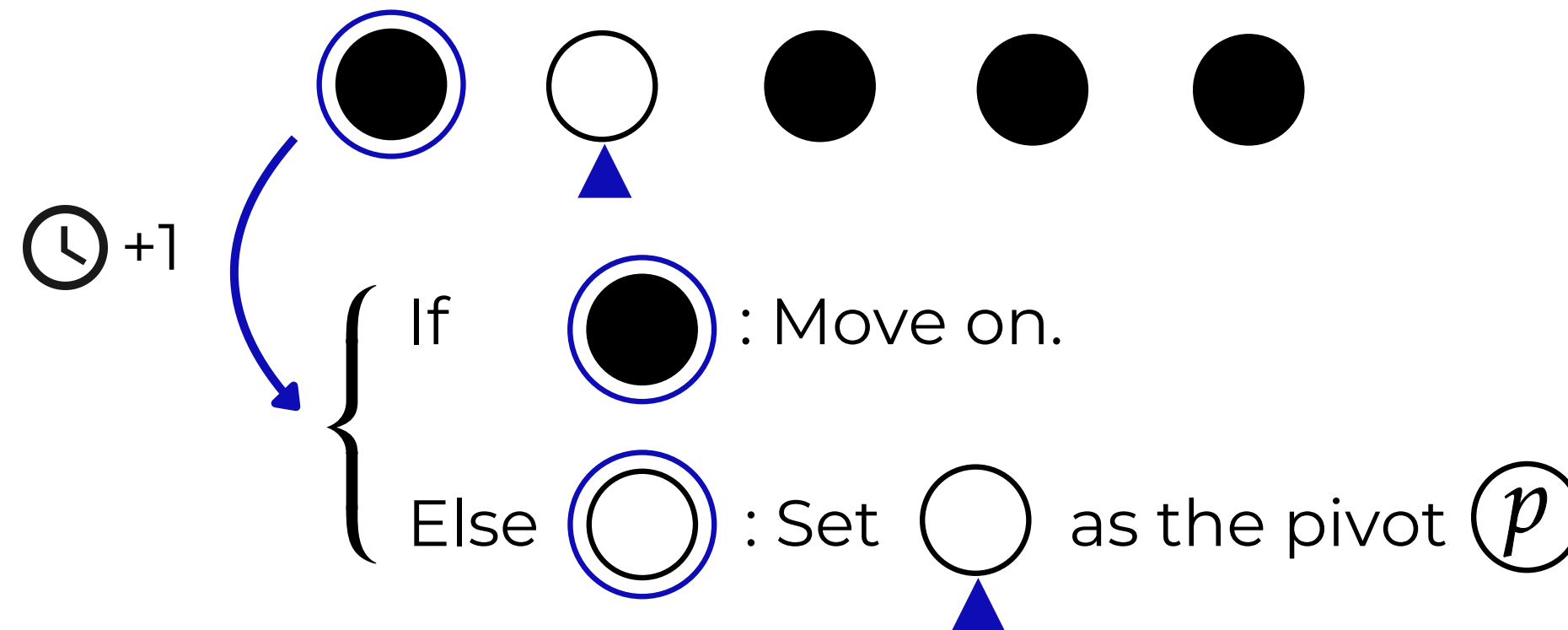
How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

Proof (sketch)

Stage 1: Find the pivot



GENERAL CASE

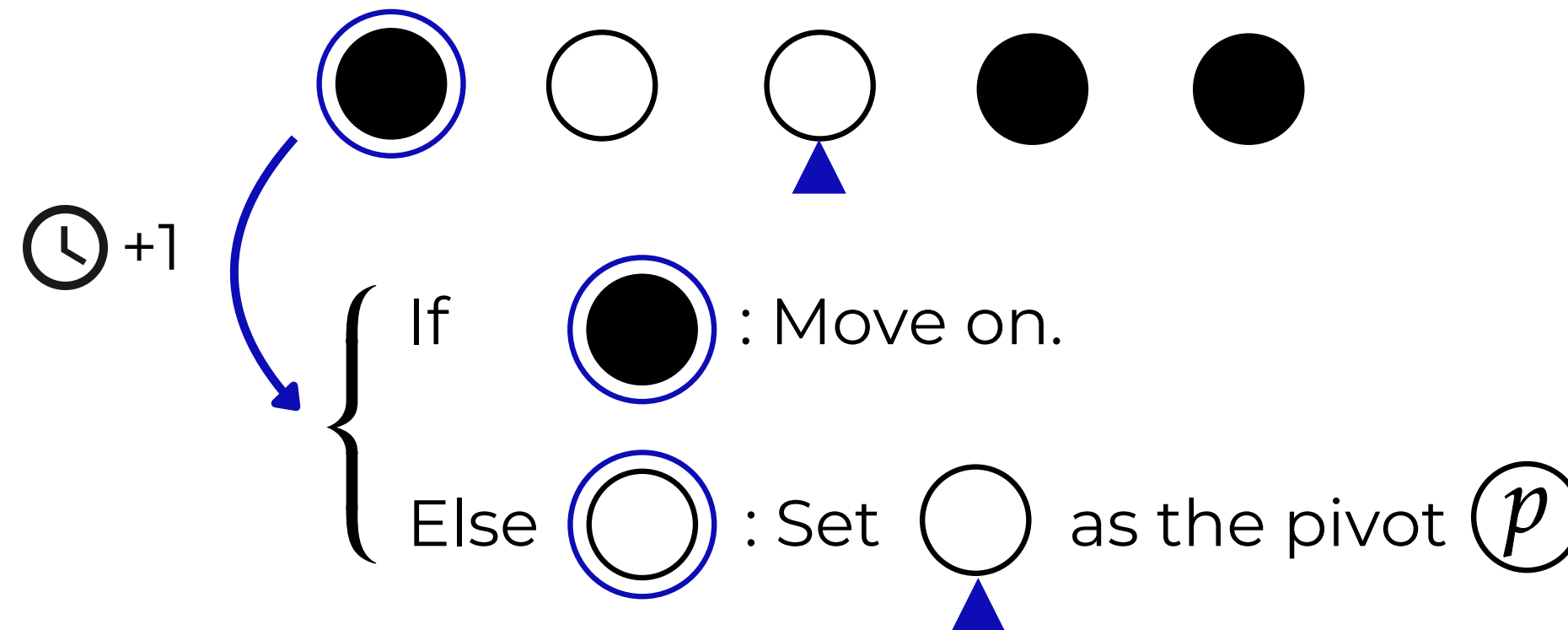
How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

Proof (sketch)

Stage 1: Find the pivot



GENERAL CASE

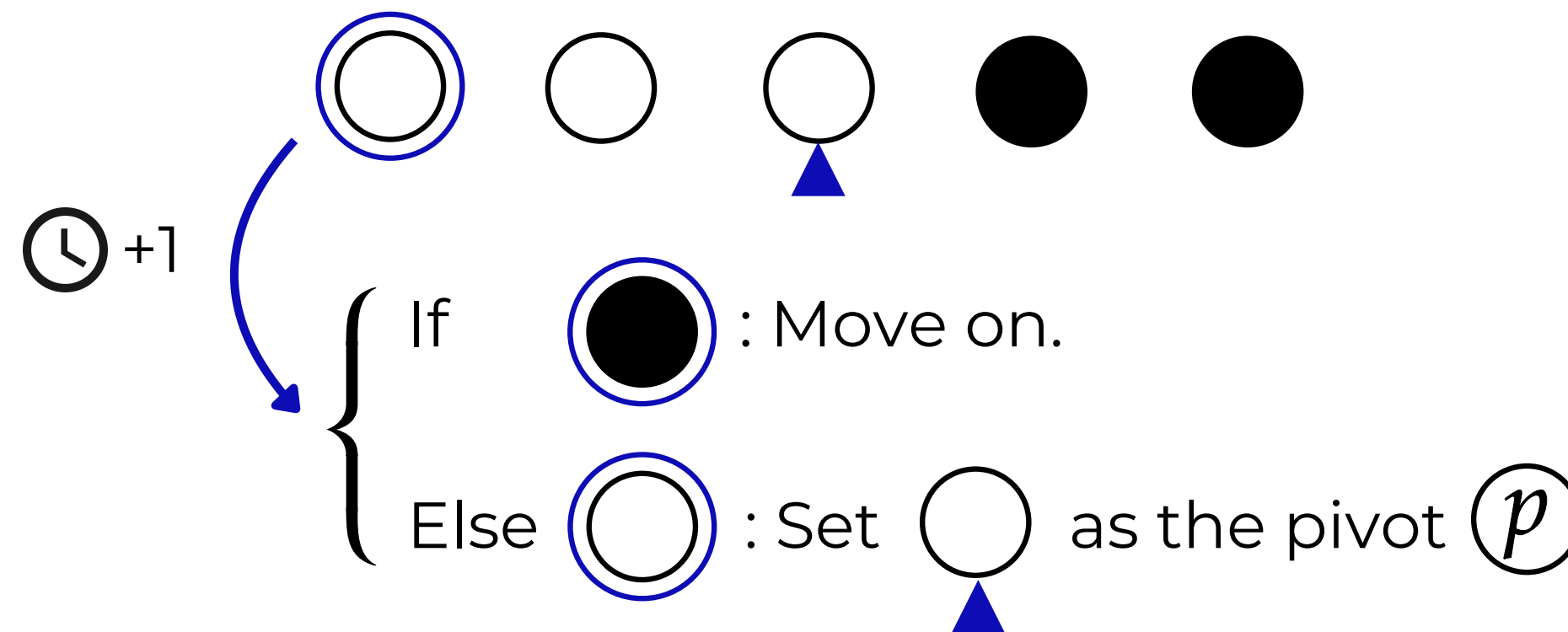
How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

Proof (sketch)

Stage 1: Find the pivot



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

Proof (sketch)

Stage 1: Find the pivot



GENERAL CASE

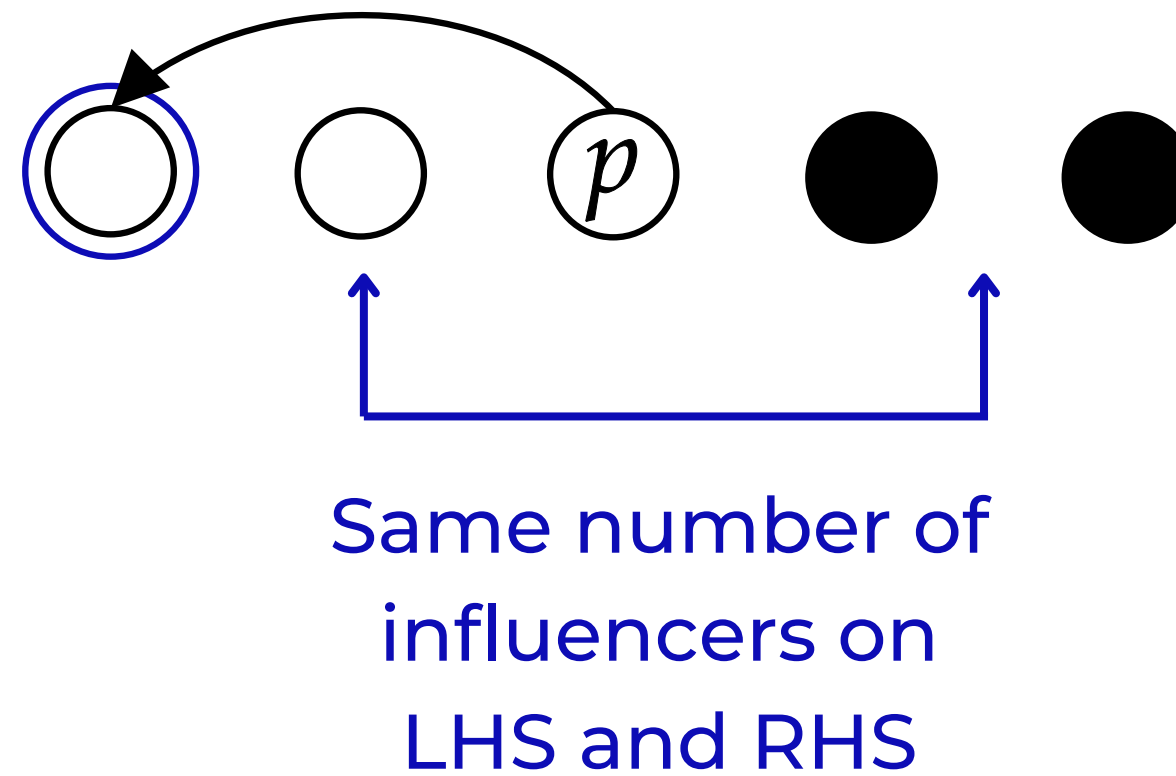
How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

Proof (sketch)

Stage 1: Find the pivot



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

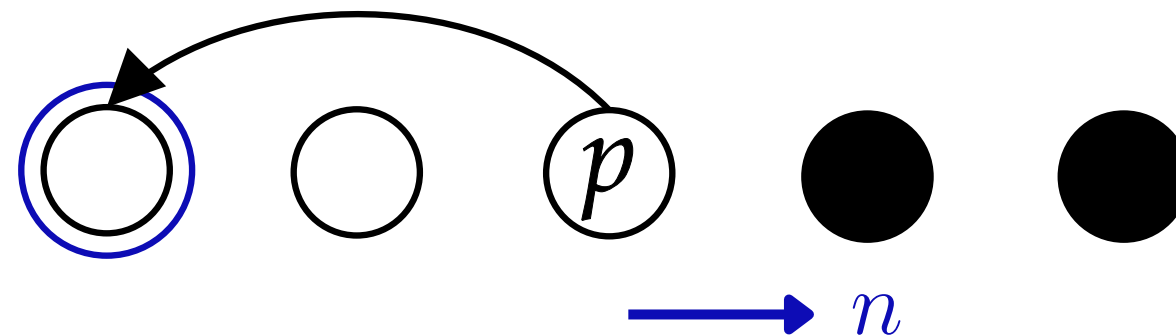
Proof (sketch)

Stage 1: Find the pivot

Budget:

$$\text{⌚} \leq n$$

$$\text{£} \leq n^2$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

Proof (sketch)

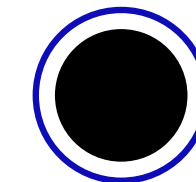
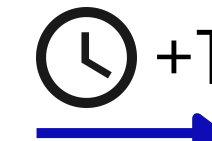
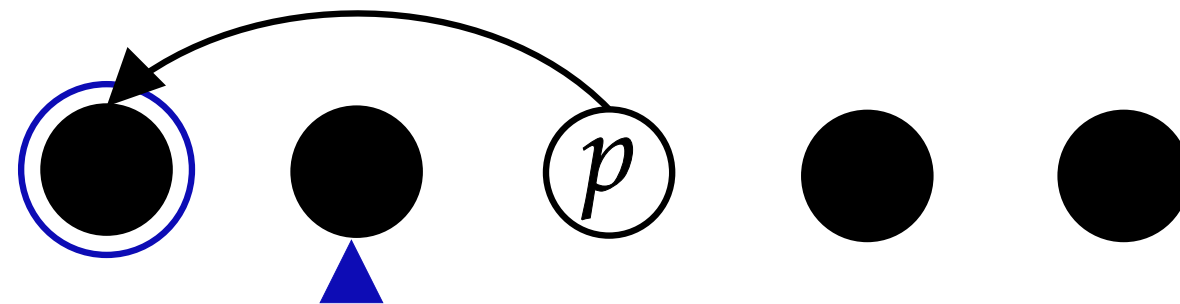
Stage 2: Find all other influencers

Budget:

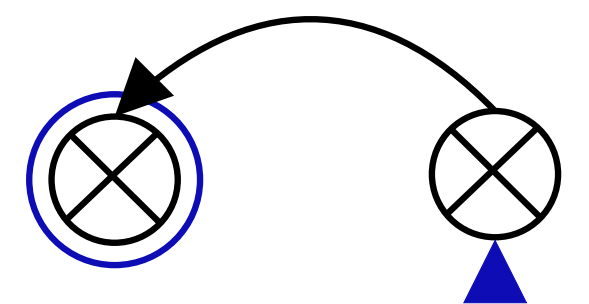
$$\text{Clock} \leq n$$

$$\text{£} \leq n^2$$

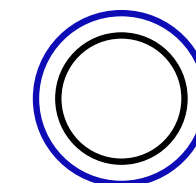
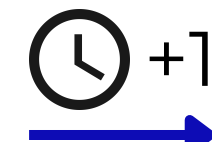
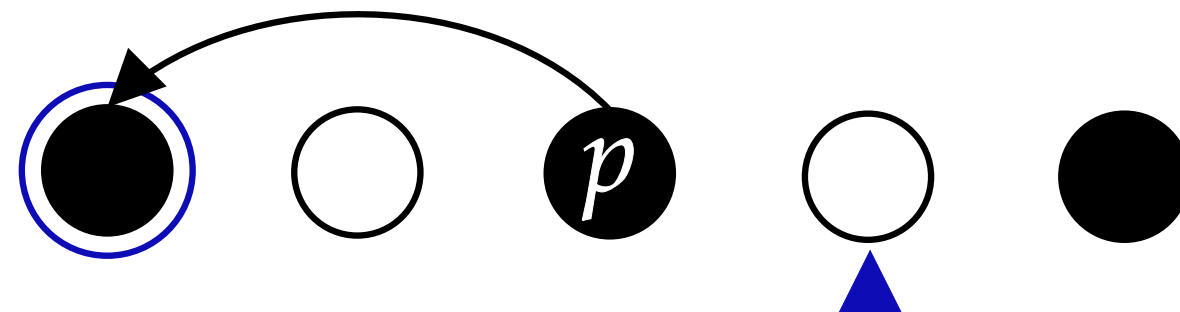
LHS



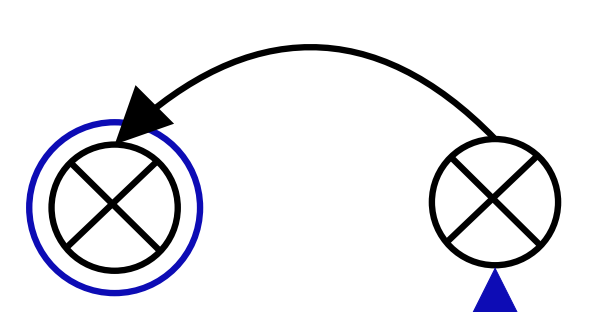
iff



RHS



iff



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

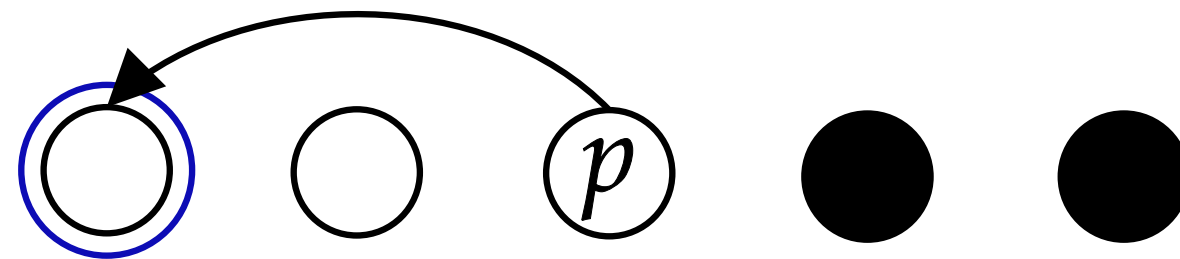
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n$$

$$\text{£} \leq n^2$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

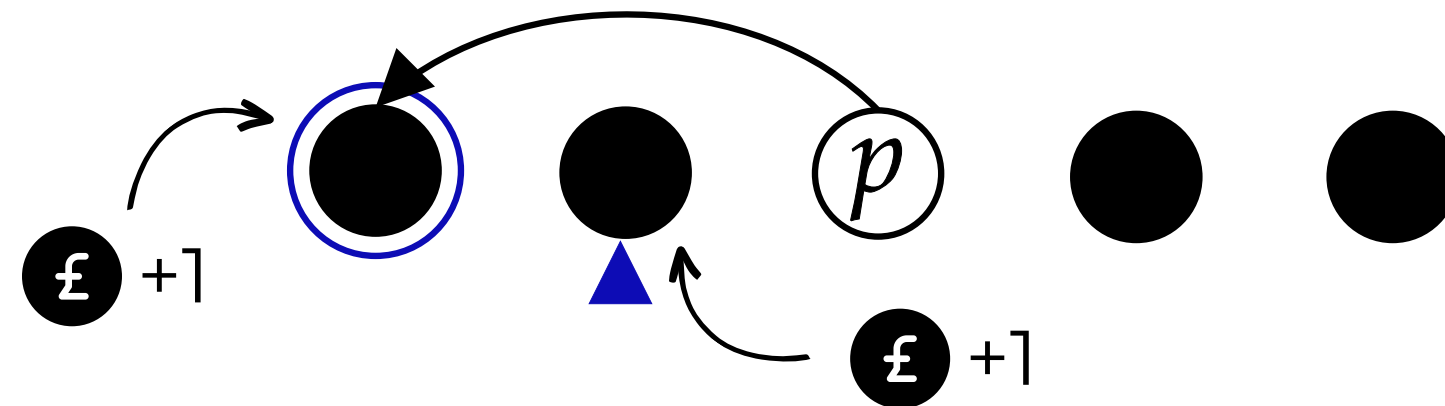
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{Clock} \leq n$$

$$\text{£} \leq n^2 + n$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

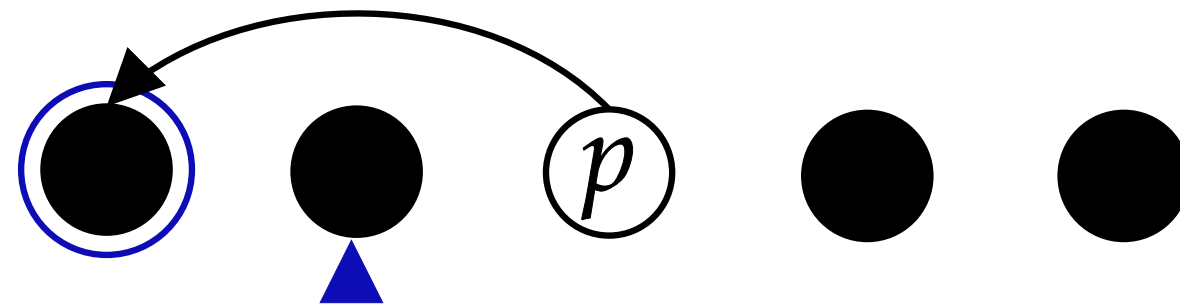
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n$$

$$\text{£} \leq n^2 + n$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

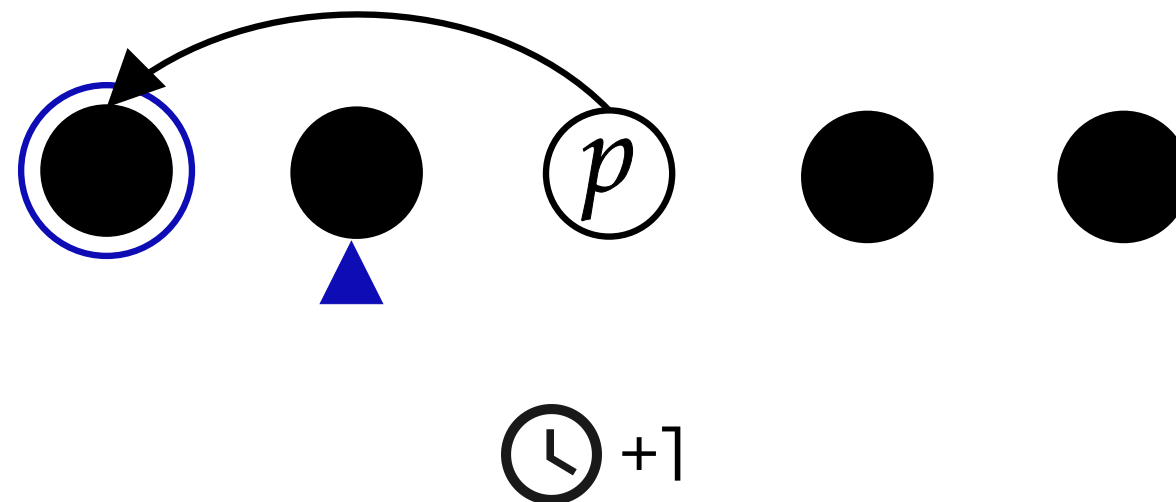
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n + 1$$

$$\text{£} \leq n^2 + n$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

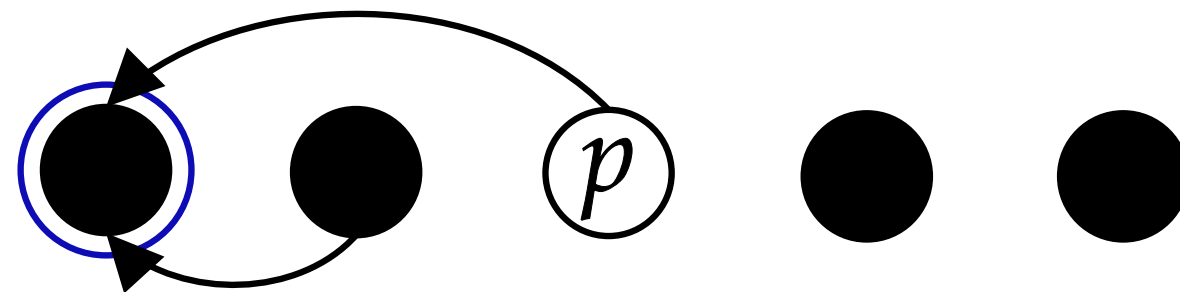
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n + 1$$

$$\text{£} \leq n^2 + n$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

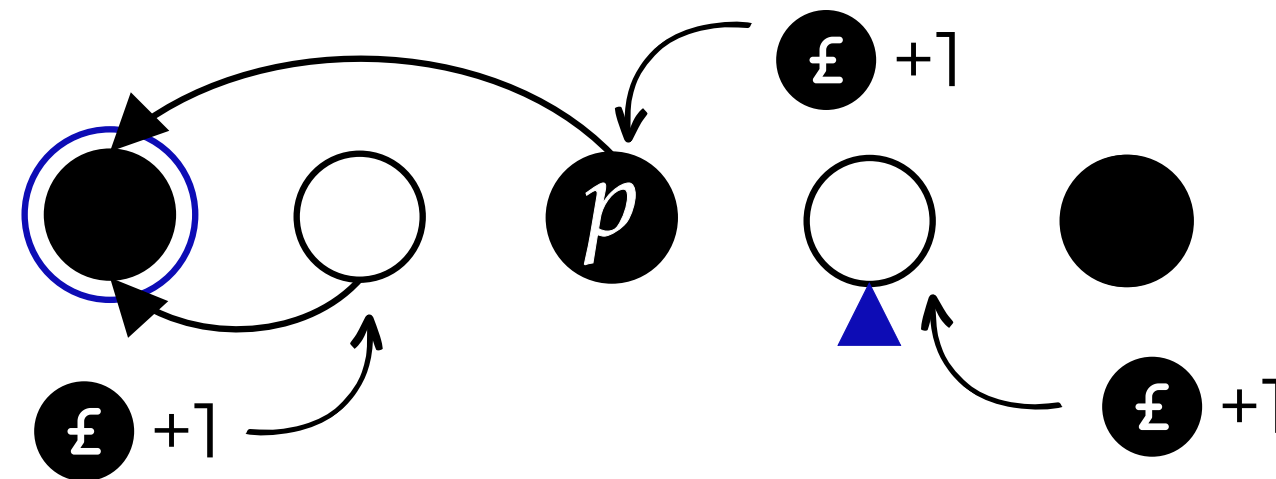
Proof (sketch)

Budget:

$$\text{Clock} \leq n + 1$$

$$\text{£} \leq n^2 + 2n$$

Stage 2: Find all other influencers



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

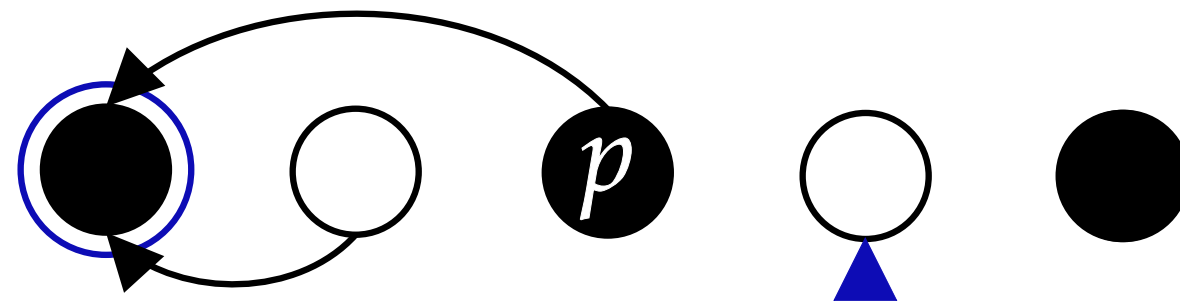
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n + 1$$

$$\text{Ⓢ} \leq n^2 + 2n$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

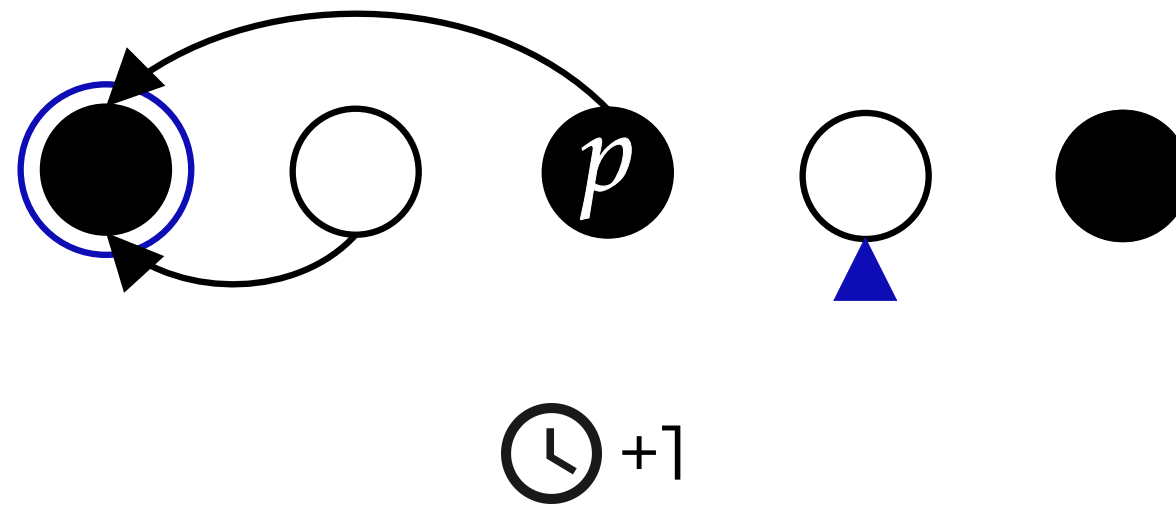
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n + 2$$

$$\text{£} \leq n^2 + 2n$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

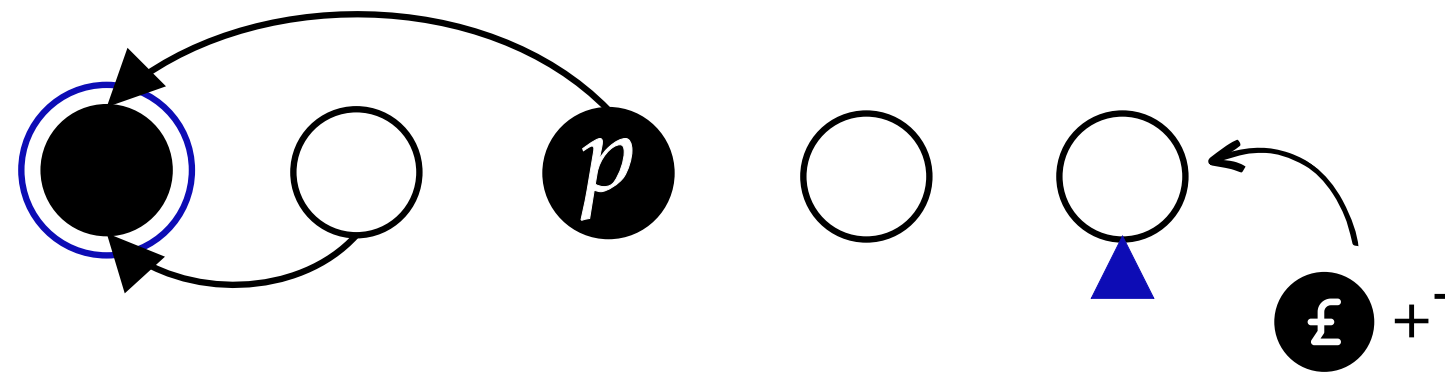
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n + 2$$

$$\text{£} \leq n^2 + 3n$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

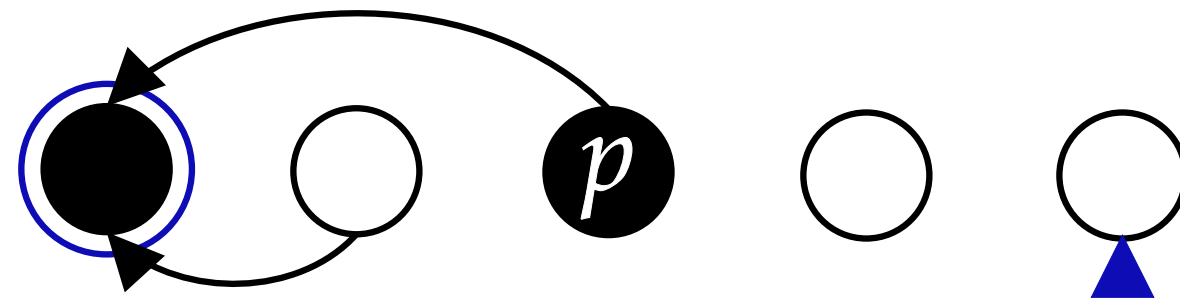
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n + 2$$

$$\text{£} \leq n^2 + 3n$$



GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

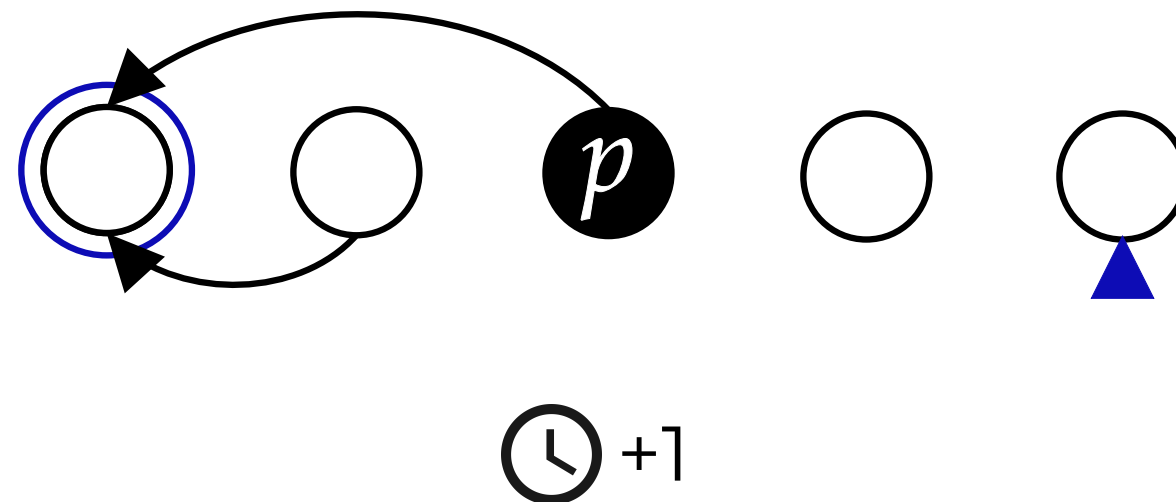
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n + 3$$

$$\text{£} \leq n^2 + 3n$$



GENERAL CASE

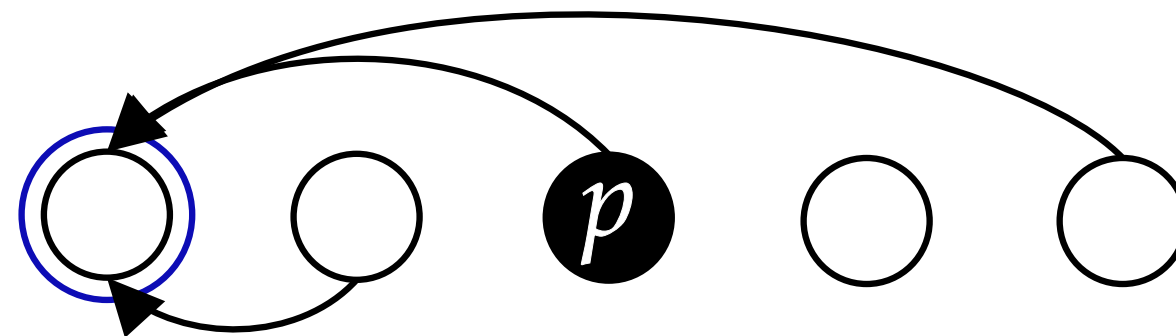
How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

Proof (sketch)

Stage 2: Find all other influencers



Budget:

$$\text{⌚} \leq n + 3$$

$$\text{£} \leq n^2 + 3n$$

GENERAL CASE

How can we identify any network?

Theorem:

A campaigner who observes a social network with n agents can learn the underlying graph exactly by using $\mathcal{O}(n^2)$ of her observation budget and $\mathcal{O}(n^3)$ of her intervention budget.

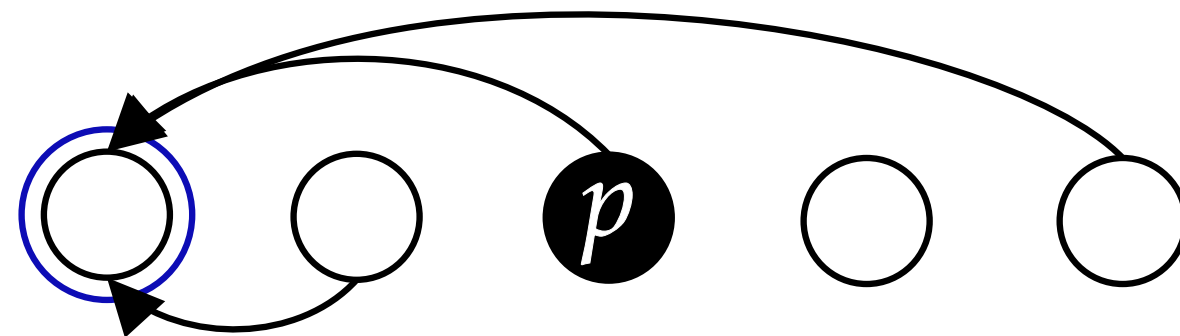
Proof (sketch)

Stage 2: Find all other influencers

Budget:

$$\text{⌚} \leq n + (n - 2)$$

$$\text{£} \leq n^2 + (n - 2)n$$



Only consistent influencers for 

SOME FOOD FOR THOUGHT

- What would happen if we had a threshold other than the majority?
- Can we efficiently learn the correct network probably approximately (PAC)?
- What would happen if we add dynamic rules to our network?
(eg. partner selection, rich get richer, ...)
- How can we scale our approach so it can handle inputs from real-world data?

