Mathematics Institute – University of Warwick

EPSRC Report

1st September 1999 - 31st October 1999
Modular Invariants, Operator Algebra and Quotient Singularities
Organisers: Miles Reid (Warwick) & David E. Evans (Cardiff)

Press release

Modular invariants, operator algebras and quotient singularities Workshop at Math Inst., Univ. of Warwick, Mon 20th-Sat 25th Sep 1999

The workshop was led by Prof. Miles Reid (Warwick) and Prof. David E Evans (Cardiff) from the UK, and Prof. Yasuyuki Kawahigashi (Tokyo Univ.) and Prof. Iku Nakamura (Hokkaido) from Japan. It set itself the task of exploring connections between different approaches to the algebra and geometry of finite subgroups of SU(2) and SU(3), including the many analogies between II_1 and III_1 subfactors in operator algebras, modular invariant partition functions in conformal field theory, finite subgroups of SU(n) and the resolution of their orbifolds. The SU(2) cases are governed by an ADE classification, and combinatorics generalising the ADE classification (for SU(3), say) appear in many different areas of recent research. The main objective of this workshop is to probe for connections to explain the empirically observed similarities between the results.

The workshop attracted many specialists in operator algebras, statistical mechanics, conformal field theory, algebraic geometry and algebra, enabling us to put on a very full program of lectures. The first half of the program featured several introductory talks: those by Gannon (modular invariants) Reid (quotient singularities) and Kawahigashi (operator algebras and subfactors). Most of the talks were attended by people from several of the different specialities, providing an appreciation of the general shape of each others' fields; for example, the algebraic geometers gained several different kinds of insight into how modular invariants relate to boundary conformal field theory.

Prose report

53 participants took part (from more than 15 countries, with around 25 UK based mathematicians, and 8 from Japan). There were 22 talks plus a Q&A session. The list of participants and the lecture program are enclosed.

It should be emphasised that one of the main concerns of the whole meeting was communication between the different perspectives. Background talks were given in three areas by Gannon (modular invariants), Reid (quotient singularities) and Kawahigashi (subfactors). There were talks by Evans and Jens Böckenhauer on their collaboration (some of which is joint with Kawahigashi) on the use of alpha induction to provide fusion graphs associated with nets of subfactors and how such subfactors provide modular invariants.

Feng Xu spoke on the subfactor approach to algebraic coset theories and related topological

quantum field theories. There is striking work in this area especially in relation to modular invariants, which has for example produced counterexamples to the Kac-Wakimoto conjecture.

Zuber, Pearce, Petkova spoke on their joint work with Behrend on thier approach to understanding ADE classifications of modular invariants via boundary conformal field theories which naturally provide representations of the Verlinde fusion rule algebras, and hence graphs whose spectra are the spectra of the fusion rule representations. Fuchs has also made fundamental contributions to the recent flurry of activity in boundary conformal field theories and gave an overview on related matters.

SU(3) analogues of ADE classifications were frequently mentioned in a variety of contexts. Ruelle discussed Fermat curves and how they appeared in the context of SU(3) modular invariants.

Evans and Jens Böckenhauer have been collaborating since 1996 on the relationship between nets of subfactor and modular invariants (with some of results obtained jointly with Kawahigashi). Since the conference they have gone on to obtain a decomposition of modular invariants obtained from subfactors into block diagonal and permutation parts - an analogue of the Moore Seiberg decomposition in conformal field theory and shown that their theory is rich enough to handle nonsymmetric invariants without the use of the Verlinde formula which depends on non-degenerately braided systems.

Batyrev, Ito, Nakamura, Reid and other algebraic geometers at the meeting have been involved in several pieces of research centring around different versions of the McKay correspondence, G-Hilbert schemes, quiver varieties, mirror symmetry, etc. In particular, Nakamura and Reid and their colleagues and students have several papers currently in progress on the McKay correspondence and G-Hilbert schemes. Several of these papers will benefit from the opportunity provided by the workshop to consult and compare notes.

Bridgeland, Crawley-Boevey, King and Nakajima discussed aspects of representations of quivers and preprojective algebras. This work relates moduli spaces, representation of algebras, and the algebraic geometry of resolutions of singularities.

Other research stimulated by this project:

Nakamura won a joint Japan-UK joint research project from JSPS for a workshop to take place at Warwick, 24th Jul-12th Aug 2000, involving at least 10 Japanese and a similar number of UK participants. Another conference is planned at Kyoto Univ,. RIMS 26th Nov-2nd Dec 2000, envisaged as a follow up to the Warwick meeting, and involving many of the same participants. For details, see www.math.sci.hokudai.ac.jp/~nakamura Reid lectured on 3-fold quotient singularities and the McKay correspondence to the Bourbaki seminar in Nov 1999, and gave a lecture series to the French CNRS Groupement de Recherche in algebraic geometry at Luminy in Dec 1999. These are among the main topics for the Newton Inst. 3-folds activity in Feb-Jul 2002, and will feature in programs of EAGER (European algebraic geometry research training network). Evans gave 5 hours of lectures on nets of subfactors and modular invariants at a summer school in Argentine in Jan 2000, attended by about 70 graduate students and postdocs. There is a strong possibilit of further work in this area in

South America with particular interest shown by research group in Brazil.

Sample references

J. Böckenhauer and D.E. Evans, Modular invariants from subfactors: Type I coupling matrices and intermediate subfactors, preprint math.OA/9911239

J. Böckenhauer and D.E. Evans, Lectures on subfactors and modular invariants. Argentine 2000, in preparation.

Tom Bridgeland, Alastair King, Miles Reid, The McKay correspondence as an equivalence of derived categories (Mukai implies McKay), preprint math/9908027, 17 pages, revised draft currently 25 pp.

Alastair Craw and Miles Reid, How to calculate A-Hilb C^3, preprint math/9909085, 29 pp.

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W. Crawley-Boevey, On the exceptional fibres of Kleinian singularities, preprint, to appear in Amer. J. Math., available from www.amsta.leeds.ac.uk/~pmtwc

Y. Gomi, I. Nakamura, K. Shinoda, Hilbert schemes of G-orbits in dimension 3, to appear in Asian J. Math.

Hiraku Nakajima, Quiver varieties and finite dimensional representations of quantum affine algebras, prepring math/9912158, 88 pp.

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Miles Reid, La correspondance de McKay, Séminaire Bourbaki, 52ème année, novembre 1999, no. 867, to appear in Astérisque 2000, preprint math/9911165, 20 pp.

Publicity:

Grant ref number GR/M64826

Grant period 1st Sep to 31st Oct 1999

Investigators: Miles Reid (Warwick) and David E Evans (Cardiff) Department: Mathematics

Inst., Univ. of Warwick

Title of project: Modular Invariants, operator algebras and quotient singularities

Objectives from the application
The workshop set itself the task of exploring connections between different approaches to the algebra and geometry of finite subgroups of SU(2) and SU(3), including the many analogies between II_1 and III_1 subfactors in operator algebras, modular invariant partition functions in conformal field theory, finite subgroups of SU(n) and the resolution of their orbifolds. The SU(2) cases are governed by an ADE classification, and combinatorics generalising the ADE classification (for SU(3), say) appear in many different areas of recent research. The main objective of this workshop is to probe for connections to explain the empirically observed similarities between the results.

Achievements of the research:
The workshop attracted many specialists in operator algebras, statistical mechanics, conformal field theory, algebraic geometry and algebra, enabling us to put on a very full program of lectures. The first half of the program featured several introductory talks: those by Gannon (modular invariants), Reid (quotient singularities) and Kawahigashi (operator algebras and subfactors). Most of the talks were attended by people from several of the different specialities, providing an appreciation of the general shape of each others' fields; for example, the algebraic geometers gained several different kinds of insight into how modular invariants relate to boundary conformal field theory.

Progress of the research:
(i) No, no change compared to the proposal
(ii) Yes, the research proceeded on time
(iii) No, no impediments

Further research
(i) No, no further EPSRC grant proposals (Dai: do you have anything to declare under this heading?)
(ii) Has the research stimulated further work in other ways? Yes. Nakamura won a joint Japan-UK joint research project from JSPS for a workshop to take place at Warwick, 24th Jul-12th Aug 2000, involving at least 10 Japanese and a similar number of UK participants. Another conference is planned at Kyoto Univ,. RIMS 26th Nov-2nd Dec 2000, envisaged as a follow up to the Warwick meeting, and involving many of the same participants. For details, see www.math.sci.hokudai.ac.jp/~nakamura Reid lectured on 3-fold quotient singularities and the McKay correspondence to the Bourbaki seminar in Nov 1999, and gave a lecture series to the French CNRS Groupement de Recherche in algebraic geometry at Luminy in Dec 1999. These are among the main topics for the Newton Inst. 3-folds activity in Feb-Jul 2002, and will feature in programs of EAGER (European algebraic geometry research training network). Evans gave 5 hours of lectures on nets of subfactors and modular invariants at a summer school in Argentine in Jan 2000, attended by about 70 graduate students and postdocs. There is a strong possibilit of further work in this area in South America with particular interest shown by research group in Brazil.

Manpower
Secretarial: two clerical assistants, Peta MacAllister and Hazel Graley were partly employed

on this project from 1st Sep 1999, and the project contributes UKL500 towards their salaries.

Collaboration? Yes/No All the workshop participants.

Expenditure
(i) Yes, 20% cut from secretarial expenses (from UKL620 to UKL500), to add to travel and living expenses
(ii) No, no change

Facilities
None, beyond the ordinary use of the Warwick Math Institute as a conference venue

Publication and dissemination of results:
(i) List of papers:

J. Böckenhauer and D.E. Evans, Modular invariants from subfactors: Type I coupling matrices and intermediate subfactors, preprint math.OA/9911239

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(ii) Have the results been conveyed to any other person or body?
Yes/No None beyond usual dissemination of results

(iii) Was there any publicity? EPSRC mentioned? Yes Poster distributed by mail to British math departments. websites: (1) Warwick website: www.maths.warwick.ac.uk/research

European alg geom website: www-euclid.mathematik.uni-kl.de/activities

Cardiff operator algebra group website: www.cardiff.ac.uk/uwcc/maths/research.html e-mail lists: (1) Several announcements were sent to the UK COW list (about 100 subscribers) and (2) the European Eager-gen list (about 550).

Oguiso sent round an announcement to an extensive list of Japanese algebraic geometers. (4) Kawahigashi sent out an announcement to a mailing list of Japanese operator algebraists having about 100 subscribers. EPSRC was prominently mentioned as the principal sponsor on all the above announcements.

Exploitation
No, no patentable commercial exploitation

Confidentiality
Yes, everything in this report is open

Detailed report
(i) Text report
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