Warwick Symposium Grant GR/M93789/01 for Stochastic Partial Differential Equations and Related Topics

Final report

October 2003

1 Overview

1.1 Organization

The Warwick Symposium for the year 2001-2002 was titled *Stochastic Partial Differential Equations and Related Topics.* The principal organisers were David Elworthy, Andrew Stuart and Roger Tribe (from Warwick), assisted by a Scientific Advisory Board: Arnaud Debussche (Rennes), Mark Friedlin (Maryland), Istvan Gyongy (Edinburgh) and Nic Krylov (Minnesota).

This grant funded two main workshops in the symposium year. A one-week workshop *Discrete and Continuous Stochastic Evolutions* in March (47 participants) concentrated on the links between discrete and continuous models, in particular the analysis of models using particle methods. A two week general workshop *Stochastic Partial Differential Equations* in August 2001 (82 participants) was followed by a ten-day period of emphasis on stochastic fluid models titled *Flows, Fluids, Filtrations and Filaments.* In addition we ran a series of 10 mini-sessions, over the period October 2000 to July 2001, on specific topics. One aim of these was to include some introductory talks in each mini-session to allow non-specialists a route into the area. These were mostly two-day events and averaged 28 registered participants. The largest of these mini-sessions *Infinite dimensional models in finance* was jointly funded by an LMS grant.

Overal 220 individuals visited one or more of the above workshops, of whom 96 were from the UK. An appeal to 20 of the key participants for papers which arose principally as a result of the symposium activity yielded a list of 35 papers (listed, along with all participants, in the full report at www.maths.warwick.ac.uk/research/reports/index.html).

There were a large number of 'long stay' participants, apart from the permanent Warwick staff with interests in the area. Jerzy Zabczyk (Warsaw) was awarded a Leverhulme Visiting Professorship to spend the whole year at Warwick. Other researchers who spent a month or more at Warwick during the symposium included Sandra Cerrai (Florence), Benadetta Ferrario (Bonn), Salah Mohammed (Carbondale), Carl Mueller (Rochester), Andrei Piatnitski (Moscow), Szymon Peszat (Krakov), Marco Romito (Pisa), Francesco Russo (Paris) and Yoshiki Otobe (Nagano). Outreach funding from this grant allowed these, and other visitors (Bloemker, Gliklikh, Hairer, Kwiecinska, Maslowski, Simao), to make research and seminar visits to other UK universities (including Edinburgh, Hull, Loughborough, Manchester, Sussex, Swansea).

Activity at Warwick in SPDE continued throughout the three years of the grant. Four postdoctoral students spent a year or more at Warwick: Dirk Blomker, Martin Hairer, Greg Pavliotis, Petter Wiberg. Stella Brassesco is spending 10 months sabbatical at Warwick during 2003-4. At the end of the three year period, the symposium grant funded a two-week workshop *Stochastic Partial Differential Equations and Related Topics* in August 2003 (65 participants) intended to review the work done since the main symposium year, and allow us to invite certain workers who were unable to come in 2000-2001. We consider this continuing activity in SPDE at Warwick one of the major outcomes of this grant.

As usual the Symposium was very efficiently organized by the staff of the Warwick Mathematics Research Centre in their customary relaxed and friendly manner, and benefitted considerably from the unique atmosphere of the Warwick Mathematics Institute.

1.2 Related activity

The symposium workshops combined with various other activity in stochastic analysis at Warwick during 2000-2003.

- Computational aspects were a recurrent theme throughout the year and we applied successfully for a separate EPSRC grant to fund a one-week workshop *Computational Stochastic Differential Equations* in March 2001.
- The UK-Japan Winter School ran in January 2001 with key speakers Terry Lyons (Oxford) and John Keating (Bristol).
- A one-week workshop on Levy Proceses (see http://science.ntu.ac.uk/msor/conf/Levy/) ran in March 2001.
- Two more mini-sessions on *Fractional Brownian motion* and *Two scale analysis of stochastic sysetms* ran in 2002 and 2003 respectively.
- Eugene Dynkin was awarded an honourary degree in 2003 and we held *Dynkin day* - a day of talks related to his research interests.
- The Warwick stochastic analysis seminar ran throughout the year.

SPDE visitors were involved in runnning, as well as participants in, all this activity.

1.3 Other funding

Other sources of funding for the symposium were:

- **London Mathematical Society** Support for UK participants via the Warwick British Visitors Fund;
- **INTRA** funding for visitors form the former Soviet Union;
- **Marie Curie Training Site** allowing five European PhD students to spend between 3 and 12 months training in Stochastic Analysis at Warwick during this period.

In addition many participants received full or partial support from their own institutions and/or national research funding agencies.

2 The Programme

2.1 Main Workshops

2.1.1 Discrete and Continuous Stochastic Evolutions: 19 - 23 March 2001

There were two main series of talks (4 each)

Tom Kurtz (Madison)	Particle representations for stochastic PDEs
Andreas Greven (Erlangen)	Longtime behaviour of interacting spatial multitype systems

The other talks given in this workshop were:

Robert Adler	Technion	The geometry of Gaussian fields on manifolds
Slava Belavkin	Nottingham	Continuous stochastics as boundary value problems in Fock space
Dan Crisan	Imperial	Exact rates of convergence for particle representations for the Zakai equation
Pierre del Moral	Toulouse	Genealogies and increasing propagations of chaos for Feynman-Kac and genetic Models
Alison Etheridge	Oxford	Survival and extinction in a locally regulated population
Klaus Fleischmann	Berlin	Clumping of a super-Brownian reactant with a stable catalyst
Jurgen Gartner	Berlin	Aspects of intermittency in the parabolic Anderson model
Geoffrey Grimmett	Cambridge	Stochastic evolution of ferromagnets
Peter Kotolonetz	Cleveland	Derivation of correlated Brownian motions from Hamiltonian systems of particles
Sylvie Meleard	Paris	A probabilistic approach to the Boltzmann equation without for non Maxwell molecules
James Norris	Cambridge	Existence and uniqueness for spatial coagulation equations
Stephano Olla	Cergy	Diffusive fluctuations in interacting particle systems
Terry Lyons	Oxford	Characterization and identification of path processes
Michael Rockner	Bielefeld	Weak Poincare inequalitites and L^2 convergence rates of Markov semigroups
Richard Sowers	Illinois	On Hamiltonian systems with small noise
Yuri Suhov	Cambridge	Convergence to equilibria and hydrodynamic limits for hyperbolic equations
Jonathon Warren	Warwick	Some simple examples of sensitivity
Shinzo Watanabe	Kyoto	Stochastic flows in duality and noises
Oleg Zaboronsky	Warwick	Statistical field theory of stochastic coalescence

2.1.2 Stochastic PDEs and Related Topics: 16 - 29 July 2001

In this general workshop the talks given were

Dirk Bloemker	Augsburg	Ginzburg-Landau Formalism for Stochastic PDEs
Stella Brassesco	Caracas	Interface fluctuations for stochastic phase field equations in $d = 1$
Zdzislaw Brzezniak	Hull	Attractors for stochastic Navier Stokes in unbounded domains
Jan van Casteren	Antwerp	Problems in semigroup theory and the HJB equation
Sandra Cerrai	Florence	The Fleming Viot operator in L^2 spaces
Jinqiao Duan	Chicago	SPDEs for geophysical fluid dynamics
Eric Vanden Eijnden	Courant Institute	Generalized flows, intrinsic stochasticity and turbulent transport
Istvan Gyongy	Edinburgh	On regularization by noise
Martin Hairer	Geneva	Exponential mixing for a stochastic PDE driven by degenerate noise
Erica Hausenblas	Salzburg	Numerical approximation of SPDE
Paul Horridge	Warwick	Stationary distributions for a noisy reaction diffusion equation
Kostya Khanin	Newton Institute	SPDEs and Burgers turbulence
Nic Krylov	Minnesota	$L^p(L^q)$ theory for stochastic PDEs
Anna Kwiecinska	Warsaw	Stabilization of PDEs by noise
Olivier Leveque	Lausanne	Hyperbolic equations driven by boundary noises
Richard Liu	Princeton	Gibbsian dynamics and invariant measures for dissipative SPDEs
Sergey Lototski	USC	Stochastic parabolic equations in domains; weighted spaces and regularity
Bohdan Maslowski	Prague	SPDE's driven by fractional Brownian motion
Jon Mattingley	Stanford	Ergodic theory for dissipative SPDEs
Carl Mueller	Rochester	A superprocess with singular mass creation
Leonid Mytnik	Technion	PDE driven by stable noise
David Nualart	Barcelona	SDEs driven by fractional Brownian motions
		Probabilistic models for vortex filaments based on fractional Brownian motion
Hans Oettinger	Zurich	Descriptions of fluctuations in nonequilibrium thermodynamics
Martin Ondrejat	Nancy	Yamada Watanabe theory in Banach spaces
Andrey Piatnitski	Moscow	Homogenization of random reaction-diffusion equations
James Robinson	Warwick	Stability of random attractors under perturbations and approximations
Michael Rockner	Bielefeld	Infinite systems of Brownian motions with singular interactions
Boris Rosovski	USC	On Krylov's L^p theory
Bjorn Schmalfuss	Merseberg	Invariant manifolds for SPDE
Wilhelm Stannat	Bielefeld	On the regularity of transition semigroups for Fleming Viot processes
Anna Talarczyk	Warsaw	Self intersection local time for Gaussian processes in $S'(R^d)$
Alexander Veretennikov	Leeds	On large deviations for SDE approximations
Aaron Yip	Purdue	Noise and uniqueness of motion by mean curvature
Jerzy Zabczyk	Warsaw	Wong Zakai approximations for a class of stochastic evolution equations
Moshe Zakai	Technion	Tangent processes
Lorenzo Zambotti	Pisa	Integration by parts on the 3-d Bessel bridge and SPDE's with reflection
Tusheng Zhang	Manchester	On Backward Stochastic Partial Differential Equations
Huiazhong Zhao	Loughborough	Random travelling waves for stochastic reaction diffusion equations

2.1.3 Diffusions, Flows, Fluids, Filtrations, and Filaments: 30 July - 10 August 2003

After the main august workshop many participants stayed on at Warwick to attend this more specialized period, especially those working on fluid models. There were two main series of 5 talks each:

Boris Rosovski (USC) Stochastic Fluid Mechanics: Stochastic diffeomorphisms and fluid dynamic; Stochastic Stokes equation; Stochastic Navier-Stokes Equations; Propagation of Gaussian Chaos by equations of fluid mechanics and moment theory.

Boris Tsirelson (Tel Aviv) Brownian motions in groups and semigroups; Filtrations in the light of general classification theory; From non-Brownian filtrations to harmonic measures; Off white noises and product systems; Stability and sensitivity on cubes and trees.

The other talks given in this period were:

Hakima Bessaih	Pisa	A mean field result for vortex filaments
Benedetta Ferrario	Bonn	Invariant measures for 2D Navier-Stokes
Yuri Gliklikh	Veronesz	Viscous hydrodynamics via stochastic processes on diffeomorphism groups
Nickolai Krylov	Minnesota	A supermartingale characterization of sets of stochastic integrals
Yves LeJan	Paris	Kraichnan turbulent advection model and stochastic flows
Marco Romito	Florence	Partial regularity for the stochastic Navier-Stokes equations
Aubrey Truman	Swansea	Singularities of Stochastic Burgers Equations and Intermittency

2.1.4 Stochastic PDEs and Related Topics: 4 - 15 August 2003

This workshop allowed us to see what work had been done since the main symposium year and to invite certain specialists who were unable to attend in 2000-2001. A new organising committee (Dirk Bloemker, David Elworthy, Martin Hairer, Andrew Stuart, Roger Tribe) reflected the long-time visitors we had during this year. The talks given in this workshop were:

Sigurd Assing	Edinburgh	On the scaling of asymmetric exclusion processes
Peter Baxendale	USC	Lyapunov exponents and stability for the stochastic Duffing - Van der Pol equation
Dirk Bloemker	Warwick	Multiscale expansion of invasriant measures near a bifurcation
Zdzislaw Brzezniak	Hull	Approximation for stochastic NSE in unbonded domains
Sandra Cerrai	Florence	Large deviations for stochastic reaction diffusion systems
Steven Evans	Berkeley	Geometry of the space of real-trees and tree-valued processes
		Markov mortality models: some implications of quasistationarity
Benedetta Ferrario	Pavia	Some uniqueness results for the 2D Navier-Stokes equation with additive noise
Yuri Gliklikh	Voronezh	On conditions for global existence of solutions of various differential equations
Giuseppina Guatteri	Milan	On the backward stochastic Riccati equation in infinite dimensions
Christoph Gugg	Stuttgart	Approximation of a stochastic Burgers equation by Markov jump processes
Martin Hairer	Warwick	Ergodicity of SDEs driven by fractional Brownian motion
Erika Hausenblas	Salzburg	Numerical approximation of the stochastic Navier Stokes
Yaozhong Hu	Kansas	Stochastic Partial Differential Equation Driven by Fractional Noise
Peter Kloeden	Frankfurt	Exponential stable stationary solutions for stochastic evolution equations
Yves Le Jan	Paris	Between coalescence and diffusion
Hannelore Lisei	Berlin	Random attractors for SPDEs
Hongwei Long	Edmonton	On Markov chain approximations to stochastic PDEs driven by Poisson measure noise
Sergey Lototsky	USC	Wiener Chaos solution of stochastic evolution equations
Grant Lythe	Leeds	Kink dynamics: from an SPDE to diffusion-limited reaction
Jonathan Mattingly	Duke	Stochastic Navier Stokes Equation: Ergodicity and Malliavin Calculus
Remigijus Mikulevicius	Vilnius	On Cauchy-Dirichlet problem for parabolic SPDEs in weighted Hoelder spaces
Salah Mohammed	Carbondale	The Stable Manifold Theorem for Semi-Linear Stochastic PDEs
Carl Mueller	Rochester	The heat equation with stationary stable noise of small index
Leonid Mytnik	Technion	SPDEs driven by stable noise
Yoshiki Otobe	Nagano	Stochastic PDEs with a type of infinitely deep square potential well
Grigorios Pavliotis	Warwick	White noise limits for inertial particles in a random field
Marco Romito	Florence	A probabilistic representation for the vorticity of a 3D viscous fluid
Tony Shardlow	Durham	Stochastic PDEs and spirals
Armen Shirikyan	Paris	Randomly forced CGL equation: stationary measures and the inviscid limit
Takis Souganidis	Austin	Homogenization for fully nonlinear PDE in stationary ergodic media
Setsuo Taniguchi	Fukuoka	Quadaratic Wiener functionals and solitons
Roger Tribe	Warwick	Stochastic comparison theorems for SPDEs
Alexander Veretennikov	Leeds	Approximations for SDEs
Anita Winter	Erlangen	Representation theorems for historical interacting Fisher-Wright diffusions
Jiang-Lun Wu	Swansea	Stochastic Burgers equation with Levy space-time white noise
Yubin Yan	Manchester	Finite element method for stochastic parabolic partial differential equations
Jerzy Zabczyk	Warsaw	Liouville theorem on harmonic functions
Lorenzo Zambotti	Pisa	Fluctuations of some interacting particle systems
Tusheng Zhang	manchester	Uniqueness of strong solutions of SDEs and SPDEs
Huaizhong Zhao	Loughborough	Stochastic elementary formula and asymptotics with caustics in one-dimension

2.2 Mini-sessions

Part of the aim of these mini-sessions were to explain problems and techniques from stochastic PDEs to non-specialists. Thus in each mini-session a speaker was invited to give one or two introductory talks. These mini-sessions usually started

at friday lunchtime and ended saturday evening to make it easier for UK workers to attend during term. They proved so popular that we extended the number we had originally planned, partly reflecting the interests of our long-term visitors.

2.2.1 Stochastic Functional Differential Equations: 10 - 11 November 2000

Saleh Mohammed	Carbondale	Stochastic Functional DEs as dynamical systems, I and II
Bernt Oksendal	Oslo	A maximum principal for controlled stochastic delay systems
		Applications of stochastic delay equations to finance
Rachel Kuske	Minnesota	Stochastic modulation equations for SDDEs
Tony Shardlow	Durham	Weak approximations of SDDEs
Xuerong Mao	Strathclyde	Attraction for solutions of SFDEs
Hannelore Lisei	Berlin	Conjugation of flows for stochastic delay equations.

2.2.2 Schrodinger Equations with Random Forcing: 17-18 November 2000

Considering the effects of random forcing, as for example in the Belavkin equation, and not the literature on Schrodinger equations with random potentials.

Thomas Zastavniak	Hull	Introduction to Feynman path integrals
		Stochastic Mehler kernels via path integration
Arnaud Debussche	Rennes	Existence and Blow-up for non-linear stochastic Schrodinger equations
Vassili Kolokoltsov	Nottingham	Quasi-classical asymptotics for the Belavkin equation
		Scattering for stochastic Schrodinger and Newton equations
Zdzislaw Brzezniak	Hull	Quantum propagators in a random metric
O. Smolianov	Moscow	Schrodinger-Belavkin equations

2.2.3 Ergodicity for spatial dynamics: 1 - 2 December 2000

In particlar one aim was comparing different methods to establish ergodicity of spatial models. This was a recurrent theme during the year which contributed later to the simultaneous breakthrough by Kuksin and Shirikyan, Mattingley and Hairer on ergodicity of various dissipative systems under suitable noises.

Yuri Kondratiev	Bonn	Gibbs measures for lattice and continuous systems: charactersiation and existence
Bogoslav Zegarlinski	Imperial	Applications of coercive inequalities to ergodicity and other problems
Andrew Stuart	Warwick	Geometric ergodicity via coupling
Jerzy Zabczyk	Warsaw	Strong Feller proceses and invaraint measures
Roger Tribe	Warwick	Coupling and comparisons - some examples

2.2.4 Stochastic Fluid Equations: 19-20 January 2001

Franco Flandoli	Pisa	The problems of singularities for 3-D fluids
		Probabilistic appraoch and results under noise perturbations
		Stochastic models of 3-D fluid vortex structures
Sergei Kuksin	Herriot Watt	Deterministic and stochastic Navier Stokes equations I
Armen Shirikyan	Herriot Watt	Deterministic and stochastic Navier Stokes equations II
Zdzislaw Brzezniak	Hull	Stochastic Euler equations
Aubrey Truman	Swansea	Singularitites of heat and Burgers equations and blow-up

2.2.5 Homogenization: 2 - 3 February 2001

Alexander Veretennikov	Leeds	Averaging for Stochastic Differential Equations I and II
Andrei Piatnitski	Moscow	Homogenization of random parabolic operators with lower order terms I and II
		Homogenization of random elliptic difference operators
Tomasc Komorowski	Pontoise	Diffusions in non-mixing Ornstein-Uhlenbeck flows
Stephano Olla	Pontoise	Bulk diffusion

2.2.6 Approximation methods for SPDEs: 16 - 17 February 2001

Andrew Stuart	Warwick	An Introduction to strong approximation for SDEs
Tony Shardlow	Durham	An introduction to weak approximation
Andrew Stuart	Warwick	An Introduction to strong approximation for SPDEs
Istvan Gyongy	Edinburgh	Approximation of SPDEs via Greens functions
Istvan Gyongy	Edinburgh	Approximation of SPDEs via L^2 theory
Jean-Sebastien Giet	Warwick	Speed of convergence for an Euler scheme for a rough functional of an SDE
Michael Tretyakov	Swansea	Weak and mean square approximations of SDEs in a bounded domain
Ben Hambly	Oxford	A Wong Zakai theorem for reversible Markov processes

2.2.7 Random Dynamical Systems: 9-10 May 2001

Hans Crauel	Exeter	Random Dynamical Systems
Bjorn Schmalfuss	Merseburg	Qualitative questions of SPDEs and climate theory
Kostya Khanin	Newton institute	Random dynamical systems and Burgers turbulence
James Robinson	Warwick	A stochastic pitchfork bifurcation in a reaction diffusion equation
Jose Langa	Seville	Attractors for stochastic partial differential equations
Hans Crauel	Exeter	Random set attractors versus random point attractors
Peter Imkellar	Berlin	Stochastic DEs as RDSs via random coordinate changes
Igor Cheushov	Berlin	Equilibria and attractors fro cooperative systems of semilinear parabolic PDEs
Szymon Peszat	Krakov	Lagrangian dynamics fro a passive tracer in a class of Gaussian markovian flows

2.2.8 Infinite dimensional models in finance: 23 - 29 May 2001

There was one preparatory series of talks by **Rene Carmona** (Princeton):

1. The Mechanics of the Fixed Income Markets; 2. Analysis of the data, and first mathematical models;
3. Infinite Dimensions; 4. Back to the Interest Rate Models: the Generalized HJM Models.

The other talks given were:

Chris Rogers	Bath	Monte Carlo valuation of American Options
Bernt Christensen	Aarhus	Infinite dimensional interest rate dynamics, stochastic volatility and yield curve calibration
Rene Cont	Paris	Term structure dynamics and parabolic SPDEs
Valdo Durlemann	Princeton	Implied correlation and spread options
Damir Filipovic	ETH	Consistency problems for Heath-Jarrow-Morton interest rate models
Saul Jacka	Warwick	No arbitrage for infinite dimensional term structure models
Marek Musiela	Paris	Pricing and risk management of derivatives written on non-traded assets
Bernt Oksendall	Oslo	Optimal control of SPDEs and applications to portfolio problems with partial observation
Josef Teichmann	Vienna	Interest rate models and infinite dimensional geometry - the classification result
Jerzy Zabczyk	Warwsaw	Variational inequalities and applications to optimal stopping

2.2.9 Stochastic Reaction Diffusion Equations: 15 - 16 June 2001

Using the outreach part of the funding this mini-session was held in Loughborough.

Sigurd Assing	Edinburgh	On the comparison method for SPDEs
Sandra Cerrai	Florence	Optimal control problems for reaction diffusion systems
Martin Hairer	Warwick	Exponential mixing for a stochastic P.D.E. driven by degenerate noise
Roger Tribe	Warwick	The KPP equation with multiplicative 'branching' noise
Huaizhong Zhao	Loughborough	Ergodic and pathwise properties of random travelling waves for the stochastic KPP equation

2.2.10 Stochastic Stability Day: 29 July 2001

Xuerong Mao	Strathclyde	Asymptotic stability in distribution of SDEs with Markovian switching
Yuhong Li	Hull	Asymptotic compactness of 2D Stochastic Navier-Stokes equations on unbounded domains
Tomas Caraballo	Seville	Asymptotic behaviour of infinite-dimensional dynamical systems perturbed by random terms