

# REPORT ON GRANT GR/S12401/01 *WARWICK SYMPOSIUM NONCOMMUTATIVE ALGEBRA AND ITS APPLICATIONS*

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## 1. BACKGROUND/CONTEXT

The EPSRC grant was a crucial instrument in carrying out the scientific programme of the 2003-2006 Warwick Symposium. It included 7 workshops over the course of the first year and 2 workshops in the subsequent two years. It is 3 more workshops than in the original proposal that demonstrates the efficiency of the execution of the project.

The activities were attended by over 200 participants, including many UK postgraduate students, with several mathematicians attending more than one. One workshop was organised outside of Warwick (in Swansea). For two of the workshops some additional funding was used.

The organisers paid for 6 long term visitors to the symposium and several long-term visitors attended with their own money.

A website <http://www.maths.warwick.ac.uk/~rumynin/na.html> for the symposium has been maintained and is still active.

## 2. THE OUTCOMES OF THE GRANT

We ran a symposium of international excellence. This is what some of the visitors have to say.

*J. Alev:* “I profited a lot from the 2 times I came to Warwick during the Symposium.

In fact, in one of them I was very interested in McKay correspondence and I could learn much about its new developments. The second time, M. Artin had asked me to give a survey talk on different things I knew about skew fields appearing in Lie theory so as to compare these with his conjecture about Noncommutative Surfaces up to birational equivalence.

I also learnt about Charu’s and Ami Braun’s results there; later I was then able to formulate a precise question about the rationality of the centre of the enveloping algebra in positive characteristics; Premet has indeed proved this for  $gl(n)$  and  $sl(n)$  in a recent paper which appeared in J. of Algebra. Further, I have now a complete Gelfand-Kirillov Hypothesis in characteristic  $p$ , proved as yet for  $gl(n)$ ,  $sl(n)$  and the Witt algebra of rank 1 by my former student J.M. Bois in a J. of Algebra paper to appear. I myself have written two papers about these subjects, one with F. Dumas which appeared in the Proceedings for the 60’th anniversary of T. Joseph, Studies in Lie theory, and the second which is now a preprint posted on Arxiv in collaboration with L. Foissy.

So I really benefited and was able to put into perspective some ideas following this symposium. If you organise another one, I will be very happy to participate.”

*V. Bavula:* “For me it was a great experience to participate in the Warwick symposium. I learnt many new things and ideas in the area of Noncommutative Algebra and around. My participation sped up finishing eight of my papers (one is listed here as [2]). I hope that in future similar symposia will continue. It was definitely a success story.”

*A. Braun:* “I found the symposium on Noncommutative algebra useful and enjoyable. It contributed to my understanding of newly investigated topics as well as affecting my own research. For instance, I am currently engaged in investigating algebras which are titled *Calabi-Yau algebras*. In my recent visit to a Warwick symposium meeting, I exchanged several results with some participants, notably Ken Brown, on this topic. As a result I managed to clean my results and was able to submit for publication a paper [4].

The big number of researchers visiting Warwick during this period helped making the symposium very interesting, and many topics which usually I am not exposed to, were presented in a rather attractive way,

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helping me to learn them in an efficient way. The connection of noncommutative algebra to algebraic geometry and *flops*, in particular, caught my attention in this symposium. To conclude I found the symposium extremely enjoyable and very useful. This shows, in my opinion, how well it was organised (in all aspects).”

*A. Elashvili:* “My contacts with the Warwick University have long history. They started from correspondence with Prof. R. Carter in the seventies. Subsequently they grew into my visits to Warwick, the longest one for three weeks (in 2004). These visits in recent years overlapped with conferences on geometric methods in noncommutative algebra and representation theory.

High scientific level of the teaching staff at the mathematics department, both the senior (Miles Reid, R. Carter) and the younger one (Dmitriy Rumynin), thorough selection of the participants for the conferences, carefulness in designing their programmes and ideal working conditions made my visits to Warwick extremely useful and substantial. Two of my recent papers - [5] started when we both were visiting Warwick, and [6] were initiated and written thanks to my visits to Warwick.”

*S. Koenig:* “The Warwick Symposium on Noncommutative Algebra has brought many benefits not only to myself, but also to my graduate students and my research assistants. The workshops provided an excellent overview over current key topics in algebra, presented by top researchers in the area. For some of my graduate students this has been a unique chance to see ‘all the famous people’. Personally, I have directly benefited not only from people and talks directly relating to my own research, but also, and in particular, from getting high level information on key topics such as categorification or symplectic reflection algebras. Moreover, at the workshops I met for the first time key researchers whom I always had hoped to meet, for example Arkhipov.”

*I. Musson:* “I visited Warwick twice in the academic year 2003-2004. The first time for the Ring Theory workshop in September 2003. The second visit was for the Noncommutative Algebra workshop in July 2004.

Both meetings were extremely helpful to my research because of the opportunities to interact with colleagues working in similar fields. In particular I would like to mention the following. During the second meeting I began discussions with Iain Gordon which eventually led to my paper [7]. ”

*L. Positselsky:* “I had a very productive visit to Warwick and mathematical conversations with Dmitriy Rumynin were of great value. I have seen significant progress in my research and I am very grateful to the organisers for inviting me.”

*A. Smoktunowicz:* “I participated at three Warwick symposiums. They benefited my research and my career in a great way. First of all, I learnt a lot about new research in my area which helped immensely with my research. Secondly, I met a lot of scientists working in my area and in other parts of algebra and was able to ask them questions and tell them about my research. I especially enjoyed talking with Charu Harajnavis, Mike Artin, Alexander Kemer, Lance Small, Chris Robson and many other scientists from UK and other countries whom I was able to meet for the first time in Warwick.

Six of my papers were influenced by my attendance at these workshops. For instance, Mike Artin whom I met at Warwick Symposium, encouraged me to work on the subject of [8] or the result of [9] was motivated by meeting professor Alexander Kemer and his work on PI algebras.

Since I previously worked in Poland I greatly appreciated the financial support especially during the first and the second Warwick meetings. I also think that participation at the Warwick meetings helped me to get a job in UK at the University of Edinburgh, since I met a lot of scientists from UK in Warwick.”

*A. Zelevinski:* “I have visited the Warwick Mathematics Institute in March - April 2004, and in particular took part in the workshop on Geometric Methods in Algebra and Representation Theory. This was a very well organised meeting, and the organisers succeeded in bringing together some of the leading experts in the field. My stay in Warwick was very beneficial for my work, I was able to make a good progress in the paper [3]. The paper ends with the following acknowledgement: *Part of this work was done while one of the authors (A.Z.) was visiting the Warwick Mathematics Institute in April 2004; he thanks Dmitriy Rumynin for his kind hospitality, and Ken Brown (Glasgow) for clarifying some issues on quantum groups.*”

### 3. DESCRIPTION OF WORKSHOPS

**3.1. LMS-EPSC advanced course Derived Categories in Algebra and Geometry, Warwick, 1-5 September 2003.** It was not a workshop but rather an instructional course for postgraduate students. It had funding from LMS. 20 graduate students working in Topology, Geometry and Algebra from UK, Hungary,

Norway, Spain and USA attended. Several mature mathematicians participated as well. The course served as an introduction to derived categories aimed at research students with broad ranging backgrounds.

There were three main lecture courses consisting of five lectures each:

Amiram Braun, *Ring Theory and Homological Algebra*,

Jeremy Rickard, *Derived Categories of Modules*,

Alastair King, *Derived Categories of Coherent Sheaves*.

The lecturers made considerable effort to coordinate material. Three two-hour problem sessions took place where students worked on problems in groups for an hour and then presented their solutions for another hour. On the final day there was a one-hour help session where everybody discussed the topic and open problems in the field. Balazs Szendroi made an exceptional job as a tutor running all the sessions.

**3.2. Ring Theory, Warwick, 8-12 September 2003.** The first workshop took place in the week following the instructional course. It was devoted to ring theory with an emphasis on some modern developments. 29 participants attended 19 lectures.

Here are some of the lecture titles.

J. Alev, *Poisson structure of some quotient varieties*,

V. Bavula, *Noncommutative affine geometry and primary decompositions for left Noetherian rings*,

A. Braun, *When is a module projective? On a question of M. Auslander*,

P. Cohn, *When are all projectives free?*,

W. Crawley-Boevey, *Multiplicative preprojective algebras and the Deligne-Simpson problem*,

A. Kemer, *On a conjecture of C. Procesi*,

M. Lorenz, *Multiplicative invariant theory*,

I. Musson, *Finite dimensional representations of invariant differential operators*,

D. Passman, *Filtrations in semisimple rings and Lie algebras*,

D. Rumynin, *Modular Lie algebras and Azumaya algebras*,

L. Small, *What goes around, comes around: 35 years of Ring Theory Problems*,

A. Smoktunowicz, *On nil rings, Jacobson radical rings and results related to Koethe's conjecture*,

J. Zhang, *Regular algebras of dimension 4*.

**3.3. Representations of Finite Dimensional Algebras University of Warwick, 8-13 December 2003.** This workshop was particularly well attended but faced a unique set of organisational problems: the Warwick Mathematical Institute was moving into a new building. The coexistence with the gang of movers was mostly peaceful but not without surprises. After the last lecture on Friday we came out for a wine and cheese reception just to find out that all the furniture was gone.

The workshop was focused on finite-dimensional algebras, their representations and some recent developments such as tilting categories. There were many graduate students among 75 participants who attended. Unfortunately Ringel and Skowronski cancelled their participation in the last moment because of personal problems.

There were 19 lectures in total. Here are some of the lecture titles:

C. Borge, *The modular isomorphism problem*,

A. Braun, *Classical orders of finite lattice type - recognition theorem*,

S. Donkin, *Some applications of tilting modules to the representation theory of symmetric groups*,

Y. Drozd, *Derived tame and wild algebras*,

K. Erdmann, *Periodic modules and Auslander-Reiten components for self-injective algebras*,

H. Krause, *The stable category of an algebraic variety*,

R. Marsh, *Cluster algebras and representation theory*,

I. Reiten, *Tilting*,

M. Saorn, *Nonexistence of large self-extensions versus finite global dimension*,

J. Schroer, *On the graph of irreducible components of module varieties*,

O. Solberg, *An application of support varieties*,

A. Tonolo, *Tilting, cotilting modules and ring extensions*.

**3.4. Geometric Methods in Algebra and Representation Theory, Warwick, 29 March - 3 April 2004.** This workshop was devoted to some new developments on the interface of geometry, algebra and representation theory such as symplectic reflection algebras, cluster algebras, representations of groups over 2-dimensional fields, etc. In total 63 participants from different disciplines attended 19 lectures.

Here are some of the lecture titles:

- H. Andersen, *Tensoring with the Steinberg module*,
- Y. Berest,  *$A_\infty$ -modules and Calogero-Moser spaces*,
- C. Coutinho, *Holomorphic foliations and nonholonomic irreducible modules*,
- V. Ginzburg, *Symplectic reflection algebras in geometry and representation theory*,
- A. Elashvili, *Classification of good gradings in simple Lie superalgebras*,
- D. Kazhdan, *Representations of reductive groups over 2-dimensional fields*,
- D. Nakano, *On the realization of orbit closures*,
- K. Rietsch, *A mirror construction for quantum cohomology of flag varieties*,
- Z. Skoda, *Noncommutative torsors and quotients*,
- S. Skryabin, *Representations of the Lie algebras of vector fields in the framework of noncommutative algebra*,
- A. Veselov, *Coxeter group actions on the cohomology of Brieskorn's pure braid groups*,
- G. Wilson, *Mad subalgebras of certain noncommutative algebras*,
- A. Zelevinski, *Double Bruhat cells and cluster algebras*.

**3.5. Noncommutative Algebra and Algebraic Geometry, Warwick, 14-18 June 2004.** This workshop drew a more geometrically minded crowd than the previous one. In fact, most of the organisational work was done by Colin Ingalls. It was cofunded by some EU money via Miles Reid. In total, 46 participants (including 8 postgraduate students) attended 15 lectures. Many world-class mathematicians attended without giving lectures: M. Artin, W. Crawley-Boevey, I. Grojnowski, I. Reiten, A. Schofield, to name a few.

Here are selected lecture titles:

- T. Stafford, *Rational Cherednik algebras and Hilbert schemes*,
- R. Buchweitz, *Hochschild cohomology and the centre of the derived category*,
- R. Bezrukavnikov, *McKay equivalence for symplectic quotient singularities*,
- T. Bridgeland *Tilting and t-structures on a non-compact Calabi-Yau*,
- J. McKay *Witten 24-dim manifold for the monster*,
- A. Yekutieli *Deformation quantization in algebraic geometry*,
- A. Kuznetsov *Derived categories of plane sections of some Fano manifolds*,
- M. Van den Bergh *Deformation Theory of Abelian Categories*.

**3.6. Hopf Algebras, Swansea 24-16 June 2004.** The workshop was held at the Department of Mathematics of the University of Wales at Swansea and attracted 29 participants from the UK, Belgium, Canada, Germany, Hungary, Israel, Italy, Mexico, Poland, Romania, Spain and the USA. The main topics of the workshop included corings and comodules, Hopf algebras, and their applications to noncommutative algebraic and differential geometry. The programme consisted of twenty-two invited and contributed talks, with longer talks at the morning and early afternoon sessions and shorter talks in the late afternoon sessions. The proceedings of the workshop were published jointly with the proceedings of the Ferrara Algebra Workshop, June 16-19, 2004 [1].

Here is a selection of the lecture titles:

- A. Ardizzoni, *Separable functors and formal smoothness*,
- S. Caenepeel, *A categorical approach to Turaev's Hopf group-coalgebras*,
- J. Gomez-Torrecillas, *Variations on comatrix corings*,
- I. Heckenberger, *Finite dimensional rank 2 Nichols algebras of diagonal type*,
- S. Majid, *Moduli spaces in finite noncommutative Riemannian geometry*,
- T. Maszczyk, *Representations of noncommutative spaces*,
- C. Menini, *Wedge products, cotensor coalgebras and formal cosmoothness in monoidal categories*,
- D. Nikshych, *Algebraic structure of fusion categories*,
- A. Rosenberg, *Quasi-coherent sheaves on noncommutative spaces*,

**3.7. Noncommutative Algebra, Warwick, 5-16 July 2004.** This was planned as the final conference but the attendance was limited probably because it was in the middle of the summer break. Overall, 36 participants attended and virtually all of them contributed to the programme of 30 1-hour lectures and 8 shorter 30-minute communications. Many interesting conversations took place over the two weeks of the conference.

Here are some of the lecture titles:

- J. Alev, *Invariants of the Weyl skew field,*
- S. Arkhipov, *Tangent spaces to monoidal categories and Lie algebra actions on Abelian categories,*
- V. Bavula, *Transcendence degree of commutative subalgebras of simple finitely generated algebras,*
- A. Braun, *Smooth PI-algebras,*
- K. Brown, *Unruffled extensions,*
- K. De Naeghel, *On the incidence between strata of the Hilbert scheme of points on the projective plane,*
- J. Gomez-Torrecillas, *Comatrix corings and reconstruction,*
- I. Gordon, *Symplectic reflection algebras ,*
- D. Hernandez, *A new fusion procedure for affinizations of quantum Kac-Moody algebras,*
- C. Ingalls, *Noncommutative surfaces with large centres,*
- S. Koenig, *Examples of relative homological algebra in representation theory,*
- L. Le Bruyn, *Brauer-Severi varieties for smooth orders,*
- S. Montgomery, *Stability of the Jacobson radical under Hopf algebra actions,*
- I. Musson, *Lie superalgebras, Clifford algebras, induced modules and nilpotent orbits,*
- H. Pfeiffer, *2-categories, 2-groups and higher gauge theory,*
- V. Retakh, *Universal algebra of pseudo-roots of noncommutative polynomials,*
- G. Roehrl, *A geometric approach to complete reducibility,*
- R. Rouquier, *Categorification and representation theory (in 2 parts),*
- D. Rumynin, *Localization for modular Lie algebras,*
- A. Smoktunowicz, *On graded and not graded domains with Gelfand-Kirillov dimension smaller than 3,*
- R. Tange, *The centres of the universal enveloping algebras of  $gl_n$  and  $sl_n$ .*

**3.8. Algebras and Representation Theory, Warwick, 26-31 May 2005.** This workshop took place 1 year later after the earlier workshops. Many new developments in algebraic representation theory were discussed at the workshop. 31 participants attended 17 lectures. Unfortunately, Roger Carter who was to give the first lecture could not take part.

Here are some of the titles:

- V. Bavula, *Generators and defining relations for the ring of differential operators on a smooth affine algebraic variety,*
- N. Cantarini, *Infinite dimensional primitive linearly compact Lie superalgebras,*
- A. Elashvili, *Hypersurface singularity and solvable Lie algebras,*
- M. Finkelberg, *Cherednik algebras in characteristic  $p$  and Hilbert schemes,*
- T. Gateva-Ivanova, *Binomial skew polynomial rings, Artin-Schelter regularity, and binomial solutions of the Yang-Baxter equation,*
- K. Goodearl, *Quantum matrices and Poisson matrix varieties,*
- I. Gordon, *Symplectic reflection algebras,*
- T. Holm, *Representation dimension of algebras,*
- D. Hernandez, *Proof of the Kirillov-Reshetikhin conjecture for quantum affine algebras,*
- V. Kac,  *$W$ -algebras and representation theory,*
- K. Kremnitzer, *Quantum gerbes,*
- K. McGerty, *The Kronecker quiver and quantum affine  $sl_2$ ,*
- A. Premet, *Quantisations of Slodowy slices and the Joseph ideal,*
- A. Skowronski, *Tame symmetric algebras with periodic modules.*

**3.9. Recent Advances in Classical Ring Theory (Goldie Memorial Meeting), Warwick, 20-24 March 2006.** The final workshop was on ring theory. It was dedicated to the memory of A. Goldie. There were 22 Participants and 16 lectures.

Here are some of the titles:

- V. Bavula, *The rings of differential operators in prime characteristic*,
- S. Launois, *Automorphisms of quantum algebras*,
- T. Lenagan, *Quantum algebras with a straightening law*,
- L. Rowen, *Recent developments in the theory of PI-rings*,
- A. Smoktunowicz, *Some results and examples on algebras with small GK dimension*.

#### 4. LONG TERM VISITORS

The organisers paid for visits of six prominent mathematicians. A. Braun visited several times and worked with C. Hajarnavis on associative algebras. L. Positselsky and A. Ulyanov visited once and worked with D. Rumynin on semi-infinite cohomology and diagonal coinvariants. A. Elashvili and V. Kac visited and worked together on good gradings. A. Elashvili visited for the second time and worked with R. Kurdiani (Leicester) and A. Premet (Manchester). A. Zelevinski visited once and worked with R. Marsh (Leeds). Besides visitors paid for by the symposium, there were several visitors, for instance, M. Artin and V. Kharchenko, who paid from their own grants.

#### 5. RESEARCH IMPACT AND BENEFITS TO THE SOCIETY

The mathematical community, in particular working in associative algebras, Lie algebras, algebraic groups, representation theory, algebraic geometry, benefited by participation in the well-run scientific program of international excellence. UK postgraduate students working in this area learnt new ideas and met prominent world mathematicians working in their area.

#### 6. EXPLANATION OF EXPENDITURE

All the money were spent on participants and on administrative support. To spend the money wisely many participants were given only a copayment with the remaining expenditure supplemented by their own grants and institutions. This allowed us to run 3 more workshops than planned.

All UK participants were given necessary support to attend. From overseas participants, we were slightly more generous to young scientists and those from Africa, Asia and Eastern Europe.

#### 7. FURTHER RESEARCH AND DISSEMINATION ACTIVITIES

A number of collaborations have started at the symposium and will definitely continue in future. Future workshops will take place on the same topics. The Swansea workshop has published its proceedings [1]. We expect that over the coming months further articles will appear in journals of international standing describing the results of research started during the symposium.

#### REFERENCES

- [1] Annali dell'Universita di Ferrara, Sezione VII, Science Matematiche, v. 51, 2005.
- [2] V. Bavula, Gelfand-Kirillov dimension of commutative subalgebras of simple infinite dimensional algebras and their quotient division algebras, *J. Reiner Angew. Math.* **582** (2005), 61-86.
- [3] A. Berenstein, A. Zelevinski, Quantum cluster algebras. *Adv. Math.* **195** (2005), 405-455.
- [4] A. Braun, On symmetric, smooth and Calabi-Yau algebras. to appear.
- [5] A. Elashvili, V. Kac, Classification of good gradings of simple Lie algebras. *Lie groups and invariant theory*, 85-104, Amer. Math. Soc. Transl. Ser. 2, 213, Amer. Math. Soc., Providence, RI, 2005.
- [6] A. Elashvili, G. Khimshiashvili, Lie algebras of simple hypersurface singularities, *J. Lie Theory* **16** (2006), 621-649.
- [7] Ian Musson, Hilbert schemes and noncommutative deformations of type A Kleinian singularities, *J. Algebra* **293** (2005), 102-129.
- [8] A. Smoktunowicz, There are no graded domains with GK dimension strictly between 2 and 3. *Invent. Math.* **164** (2006), 635-640.
- [9] A. Smoktunowicz, On structure of domains with quadratic growth. *J. Algebra* **289** (2005), 365-379.