WARWICK
THE UNIVERSITY OF WARWICK

TMUA 2016, Paper 1, Q18

18 The function $\frac{1-x}{\sqrt[3]{x^2}}$ is defined for all $x \neq 0$.

The complete set of values of x for which the function is decreasing is

A $x \leq -2,\ x > 0$

B $-2 \leq x < 0$

C $x \leq 1,\ x \neq 0$

D $x \geq 1$

E $-2 \leq x \leq 1,\ x \neq 0$

F $x \leq -2,\ x \geq 1$

The values of x for which a function is decreasing is equivalent to the values of x at which the gradient is negative

We will find the derivative function $\frac{dy}{dx}$ and identify all points of it's domain, x, for which it is negative.

Let $y = \frac{1-x}{\sqrt[3]{x^2}}$, $x \neq 0$

$$\iff \quad y = (1-x)x^{-2/3}$$

using the product rule, I have

$$\frac{dy}{dx} = (1-x)\left(\frac{-2x^{-5/3}}{3}\right) + x^{-2/3}(-1)$$

$$= x^{-5/3}\left(\frac{2x}{3} - \frac{2}{3}\right) - x^{-2/3}$$

$$= x^{-5/3}\left(\frac{2x}{3} - \frac{2}{3} - x^{(-2/3+5/3)}\right)$$

using laws of indices

$$= x^{-5/3}\left(-\frac{2}{3} - \frac{x}{3}\right)$$

This part is a reciprocal function which I know the general shape of

This part is linear and has a value of zero when $x = -2$

x	$x<-2$	$x=-2$	$-2<x<0$	$x=0$	$x>0$
$x^{-5/3}$	$-$	$-$	$-$	$*$	$+$
$-\frac{2}{3} - \frac{x}{3}$	$+$	0	$-$	$-$	$-$
$\frac{dy}{dx}$	$-$	0	$+$	$*$	$-$

so the answer is A