WARWICK
THE UNIVERSITY OF WARWICK

TMUA 2016, Paper 1, Q2

2 The expression $3x^3 + 13x^2 + 8x + a$, where a is a constant, has $(x + 2)$ as a factor.

Which one of the following is a complete factorisation of the expression?

A $(x + 2)(x - 1)(3x - 2)$

B $(x + 2)(x + 1)(3x - 2)$

C $(x + 2)(x + 1)(3x + 2)$

D $(x + 2)(x - 3)(3x + 2)$

E $(x + 2)(x + 3)(3x - 2)$

F $(x + 2)(x + 3)(3x + 2)$

In this solution we will use options A-F to our advantage. Firstly, find the constant term

$(x+2)$ is a factor $\Leftrightarrow f(-2)=0$

$3(-2)^3+13(-2)^2+8(-2)+a=0$

$-24+52-16+a=0$

$a=-12$

Now, to get this constant term, the product of the constant terms in the 3 factors must be -12

For A this gives $2\times-1\times-2=4$ so the answer cannot be A

similarly, the answer cannot be B, C or F.

For D, I have $2\times-3\times2=-12$ so the answer could be D

For E, I have $2\times3\times-2=-12$ so the answer could be E

We can see by observation that both D and E give the correct x^3 term. Let's check the x^2 term. We are looking for $13x^2$

To generate the x^2 term, I need all combinations of an x term from 2 of the factors (let's call those X) and a constant term from the other (let's call that C)

For D, I have

XXC
XCX
CXX

$(x+2)(x-3)(3x+2)$

so the x^2 term for D is $2x^2-9x^2+6x^2=-x^2$

For E I have

XXC
XCX
CXX

$(x+2)(x+3)(3x-2)$

so the x^2 term for E is $-2x^2+9x^2+6x^2=13x^2$

Therefore, the correct answer must be E