WARWICK
THE UNIVERSITY OF WARWICK

5 Consider the following three statements:

1. $10p^2 + 1$ and $10p^2 - 1$ are both prime when p is an odd prime.
2. Every prime greater than 5 is of the form $6n + 1$ for some integer n.
3. No multiple of 7 greater than 7 is prime.

The result $91 = 7 \times 13$ can be used to provide a counterexample to which of the above statements? $91 = 7 \times 13$ tells us 91 is not prime and, in particular that 91 is a multiple of 7

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

Let's consider how the "result" relates to each statement. This can be done in order or you may have a sense of which ones are more accessible to deal with
We begin with statement 3

Statement 3: Any multiple of 7 that is greater than 7 will not be prime, therefore this statement is true so no counter example could exist. We rule out D, F, G and H.

Statement 2: A counterexample to this statement must be a prime number greater than 5 that can<u>not</u> be written in the form $6n+1$, for any n.
$91 = 6(15)+1$ might seem that it relates to this statement but, in fact, it does not because 91 is not prime and this statement only relates to prime numbers. We further rule out E and C.

Statement 1: Neither 7 nor 13 can be written in the form $10p^2+1$ or $10p^2-1$ where p is an integer, let alone prime.
91 can be written as $10p^2+1$, where $p=3$
3 is an odd prime and, because 91 is <u>not</u> prime, the result $91 = 7 \times 13$ does provide a counterexample

Therefore, the answer is B.