11 The line $y = mx + 5$, where $m > 0$, is normal to the curve $y = 10 - x^2$ at the point (p, q).

What is the value of p?

A $\frac{\sqrt{2}}{6}$

B $-\frac{\sqrt{2}}{6}$

C $\frac{3\sqrt{2}}{2}$

D $-\frac{3\sqrt{2}}{2}$

E $\sqrt{5}$

F $-\sqrt{5}$

To find the gradient of $y=10-x^2$ at (p,q) I first differentiate

$$\frac{dy}{dx} = -2x$$

substituting $x=p$ gives $-2p$ as the gradient of y at (p,q)

Now m is the negative reciprocal of this

i.e. $m = \frac{1}{2p}$

The point (p,q) lies on both the line and the curve, so I have

$q = mp + 5$ and $q = 10 - p^2$ $(*)$

$q = \frac{p}{2p} + 5$

$q = \frac{11}{2}$

Substituting this value of q into $(*)$ gives $\frac{11}{2} = 10 - p^2$

$$p^2 = \frac{9}{2}$$

and since $m>0$ and $m = \frac{1}{2p}$, p must be positive

therefore $p = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$ so the answer is C.