TMUA 2018, Paper 1, Q5

5 The function f is defined by $f(x) = x^3 + ax^2 + bx + c$.

a, b and c take the values 1, 2 and 3 with no two of them being equal and not necessarily in this order.

S1: The remainder when $f(x)$ is divided by $(x + 2)$ is R.

S2: The remainder when $f(x)$ is divided by $(x + 3)$ is S.

What is the largest possible value of $R - S$?

A -26

B 5

C 7

D 17

E 29

An implication of the factor theorem tells us that if a polynomial function $f(x)$ has remainder r when divided by $(x-a)$, for any a, then $f(a) = r$

This means that statements S1 and S2, as labelled above, give the following results

$$f(-2) = R \quad \text{and} \quad f(-3) = S$$

That is, $(-2)^3 + a(-2)^2 + b(-2) + c$ and $(-3)^3 + a(-3)^2 + b(-3) + c$

$-8 + 4a - 2b + c$ and $-27 + 9a - 3b + c$

This gives an expression for $R-S$ in a, b and c as follows

$$R - S = -8 + 4a - 2b + c - (-27 + 9a - 3b + c)$$

$$R - S = 19 - 5a + b$$

To get the largest value for $R-S$, I need $a = 1$ and $b = 3$ which gives $R - S = 19 - 5(1) + 3 = 17$

and the answer is D