

TMUA 2021 Paper 1 Question 12

\* The minimum value of the function  $x^4 - p^2x^2$  is  $-9$

$p$  is a real number.

Find the minimum value of the function  $x^2 - px + 6$

A  $-3$   
 B  $6 - \frac{3\sqrt{2}}{2}$   
 C  $\frac{3}{2}$   
 D  $3$   
 E  $\frac{9}{2}$   
 F  $6 + \frac{3\sqrt{2}}{2}$

Completing the square will expose the minimum value, I have

$$x^2 - px + 6 = \left(x - \frac{p}{2}\right)^2 - \frac{p^2}{4} + 6$$

note: this part of the function is always positive so the minimum value of the function occurs when  $x - \frac{p}{2} = 0$  that is when  $x = \frac{p}{2}$

so the minimum value of this function is this expression

$$-\frac{p^2}{4} + 6$$

and the remaining step is to find  $p$

Using the information given on line \* of the question, I have

$$\begin{aligned} x^4 - p^2x^2 &= (x^2)^2 - p^2x^2 \\ &= \left(x^2 - \frac{p^2}{2}\right)^2 - \frac{p^4}{4} \end{aligned}$$

here we expose the fact that this is a quadratic in  $x^2$

here we complete the square on this quadratic in  $x^2$

so the minimum value of this function is  $-\frac{p^4}{4}$  and I have

$$\begin{aligned} -\frac{p^4}{4} &= -9 \\ p^4 &= 36 \\ p^2 &= \pm\sqrt{36} \\ p^2 &= \pm 6 \end{aligned}$$

note: it's more efficient at this stage to find  $p^2$ , rather than  $p$  because the expression I have for my minimum value has  $p^2$

Now  $p^2 = 6$  yields the lower value for the expression

$$-\frac{p^2}{4} + 6$$

that is  $-\frac{(6)}{4} + 6 = \frac{9}{2}$

note: it is well worth noticing that  $p^2 = -6$  is not valid as, in this case we would have  $p = \pm\sqrt{6}i$  and  $p$  must be a real number.

so the correct answer is  $E$