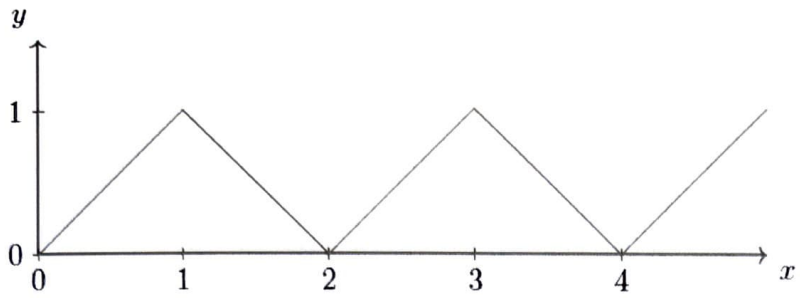
TMUA 2021 Paper 1 Q15

The diagram shows the graph of $y = \mathrm{f}(x)$.



The graph consists of alternating straight-line segments of gradient 1 and -1 and continues in this way for all values of x.

The function g is defined as

$$\mathrm{g}(x) = \sum_{r=1}^{10} f\left(2^{r-1}x\right)$$

Find the value of

$$\int_0^1 \mathrm{g}(x)\,\mathrm{d}x$$

A $\frac{1023}{1024}$

B $\frac{1023}{512}$

C 5

D 10

E $\frac{55}{2}$

F 55

To get a feel for these "f functions" see TABLE 1 which shows $f(2^{r-1}x)$ for the first 3 values of r, their respective graphs, and the value of the shaded area for each.

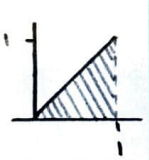
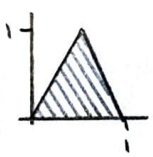
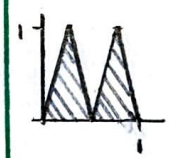
r	1	2	3	...
$f(2^{r-1}x)$	$f(x)$	$f(2x)$	$f(4x)$	...
graph between 0 and 1				...
area of shaded region	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	...

TABLE 1

note: we can evaluate these areas by observation as follows: imagine a unit square with co-ordinates $(0,0)$ $(0,1)$, $(1,1)$ and $(1,0)$. The area of each shaded region is half of the area of this unit square.

This indicates a pattern in which each area is $\frac{1}{2}$, which is in fact the case for each value of r.
Since there are 10 values of r, the sum of these areas is $10 \times \frac{1}{2} = 5$.
This is equivalent to the value of $\int_0^1 g(x)\,dx$

so the correct answer is option C.