

TMUA 2021 Paper 1 Question 16

Consider the expansion of

$$(a + bx)^n$$

* The third term, in ascending powers of x , is $105x^2$

** The fourth term, in ascending powers of x , is $210x^3$

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Find the value of $\frac{a}{b}$

A $\frac{1}{4}$ Line * tells us that $105x^2$ is the 3rd term
 Lines ** and *** tell us that $210x^3$ is the term in the middle of the expansion

B $\frac{4}{9}$ i.e. $(a+bx)^n = \dots + \dots + 105x^2 + 210x^3 + \dots + \dots + \dots$

C $\frac{25}{36}$ From this information we can deduce that $n=6$

D $\frac{5}{6}$ Now we have

E 1 $(a+bx)^6 = a^6 + \binom{6}{1}a^5bx + \binom{6}{2}a^4b^2x^2 + \binom{6}{3}a^3b^3x^3 + \dots$

Using the 3rd term here with Line *, gives

$$\binom{6}{2}a^4b^2x^2 = 105x^2$$

$$\Leftrightarrow 15a^4b^2 = 105$$

$$\Leftrightarrow a^4b^2 = 7$$

Using the 4th term here with Line ** or ***, gives

$$\binom{6}{3}a^3b^3x^3 = 210x^3$$

$$\Leftrightarrow 20a^3b^3 = 210$$

$$\Leftrightarrow a^3b^3 = \frac{21}{2}$$

Now $\frac{a^4b^2}{a^3b^3} = \frac{a}{b}$ so $\frac{a}{b} = \frac{7}{\frac{21}{2}} = \frac{2}{3}$

$\binom{6}{2} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{(2 \times 1)(4 \times 3 \times 2 \times 1)} = \frac{6 \times 5}{2} = 15$
 For integers n and r $\binom{n}{r} = \frac{n!}{r!(n-r)!}$
 and is read as "n choose r"

To find $\left(\frac{a}{b}\right)^2$ we calculate

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$$

so the correct answer is B