

TMUA 2021 Paper 1 Question 17

Which of the following sketches shows the graph of

$$\sin(x^2 + y^2) = \frac{1}{2}$$

where $x^2 + y^2 \leq 8\pi$?

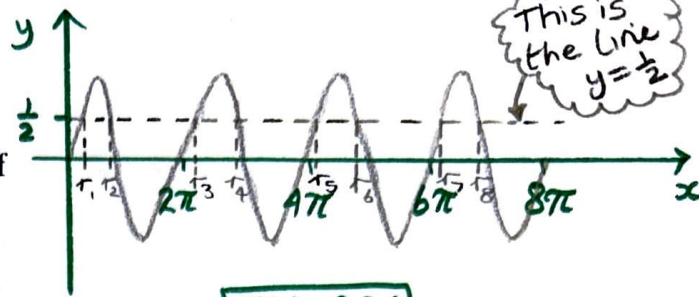
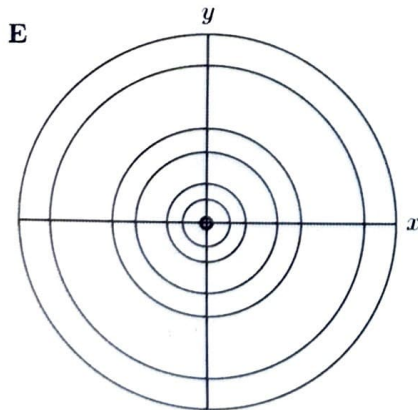
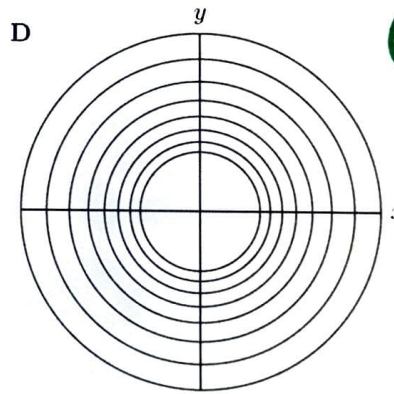
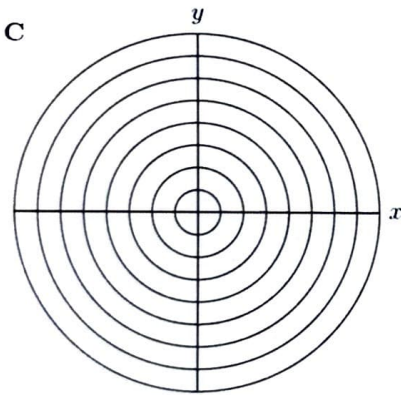
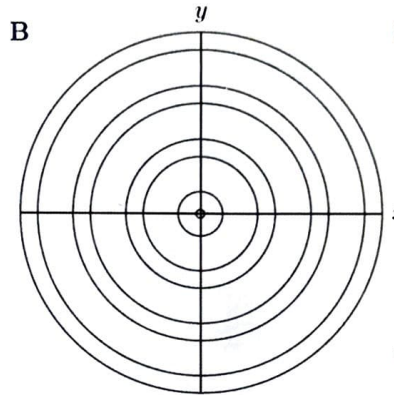
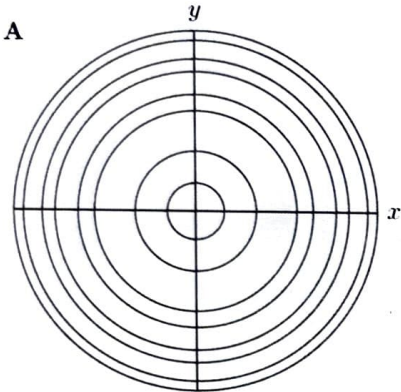


FIGURE 1

FIGURE 1 shows the graph of the sine function for positive inputs up to 8π

As the graph hits the line $y = \frac{1}{2}$, we get solutions to $\sin(x^2 + y^2) = \frac{1}{2}$

(labelled r_1, r_2, \dots, r_8 in FIGURE 1)

$x^2 + y^2 = r^2$ describes a circle, centred at the origin, with radius r .

So we can think of these r_1, r_2, \dots, r_8 as the squares of the radius' of a number of concentric circles.

From FIGURE 1, we see that they seem to emerge in pairs, equally spaced.

In order to find the actual radius' of these circles, we would need to apply the positive square root.

FIGURE 2 shows the graph of $y = \sqrt{x}$ and gives a visual demonstration to show that these radius' are in pairs

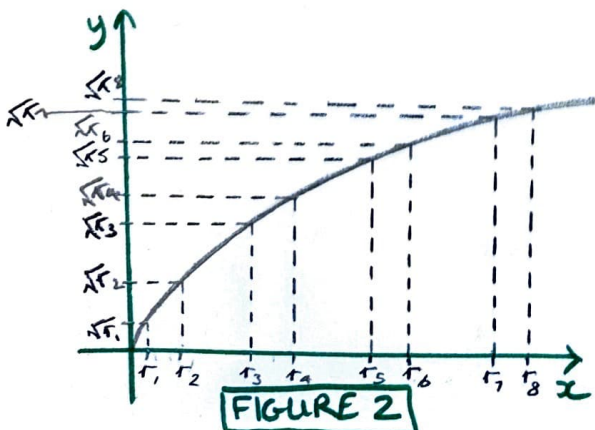


FIGURE 2

with the gap between each pair reducing, as they increase in value.

If we take the values on the y-axis in FIGURE 2 as the radii of 8 concentric circles, this would give sketch A

so the correct answer is A.