TMUA 2021 Paper 1 Q18

The curve with equation

$$x = y^2 - 6y + 11$$

is rotated 90° clockwise about the point P to give the curve C.

P has x-coordinate -2 and y-coordinate 3.

What is the equation of C ?

A $y = -x^2 - 4x - 3$

B $y = -x^2 - 4x - 5$

C $y = -x^2 - 6x - 7$

D $y = -x^2 - 6x - 11$

E $y = x^2 - 4x + 5$

F $y = x^2 + 4x + 3$

G $y = x^2 - 6x + 11$

H $y = x^2 + 6x + 7$

If I "swap" x's and y's I get the equation of a quadratic $y = x^2 - 6x + 11$

Completing the square, I get

$y = (x-3)^2 + 2$

So this quadratic has a minimum at (3, 2) - see FIGURE 1

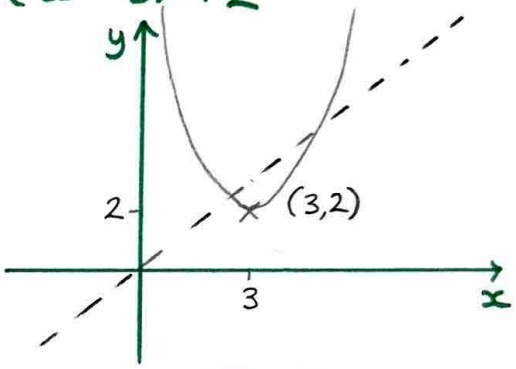

FIGURE 1

Reflecting this quadratic in the line $y=x$ will give the graph of the curve that is being rotated - see FIGURE 2, This is a parabola with vertex at (2, 3)

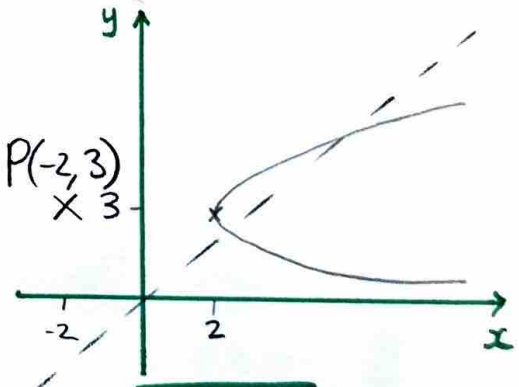

FIGURE 2

We can imagine rotating this vertex 90° clockwise about the point P, which will give curve C - see FIGURE 3

This is an "n-shaped" quadratic which will have -1 as the coefficient of x^2 and a maximum at $(-2,-1)$. We can use this information to form and expand its equation

$y = -(x+2)^2 - 1$

$y = -(x^2 + 4x + 4) - 1$

$y = -x^2 - 4x - 5$

So the correct answer is option B

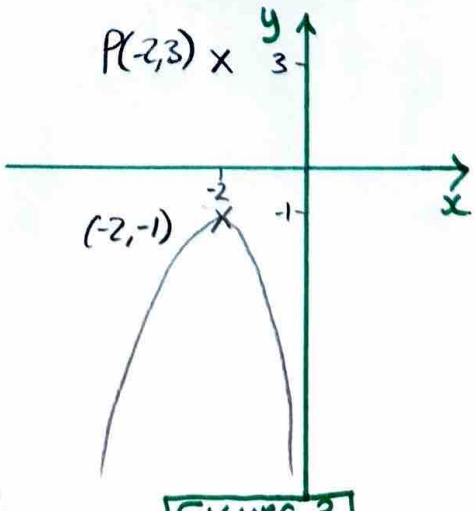

FIGURE 3