

TMUA 2021 Paper 1 Q19

The equation

$$\sin^2(4^{\cos\theta} \times 60^\circ) = \frac{3}{4}$$

has exactly three solutions in the range $0^\circ \leq \theta \leq x^\circ$

What is the range of all possible values of x ?

- A $90 \leq x < 120$
- B $90 \leq x < 270$
- C $120 \leq x < 240$
- D $270 \leq x < 300$
- E $300 \leq x < 360$
- F $450 \leq x < 630$

Let $\alpha = 4^{\cos\theta} \times 60^\circ$

Then $\sin^2 \alpha = \frac{3}{4}$

so $\sin \alpha = \pm \frac{\sqrt{3}}{2}$

By drawing a graph of the sine function for positive inputs - see FIGURE 1, we can see that α can take a number of values, namely:

$$\alpha = 60^\circ, 120^\circ, 240^\circ, 300^\circ, 420^\circ, \dots$$

and with $\alpha = 4^{\cos\theta} \times 60^\circ$, I have

$$4^{\cos\theta} \times 60^\circ = 60^\circ, 120^\circ, 240^\circ, 300^\circ, 420^\circ, \dots$$

$$4^{\cos\theta} = 1, 2, 4, 5, 7, \dots$$

$$\cos\theta = 0, \frac{1}{2}, 1.$$

note the full stop here because values greater than 4 are not valid due to $-1 \leq \cos\theta \leq 1$

Therefore, solutions will be found for values of θ at which the function meets the x-axis or either of the dotted lines in FIGURE 2. The first 3 occur when $0^\circ \leq \theta \leq 90^\circ$ and the 4th occurs when $\theta = 270^\circ$. Therefore $90^\circ \leq x < 270^\circ$

so the correct answer is option B

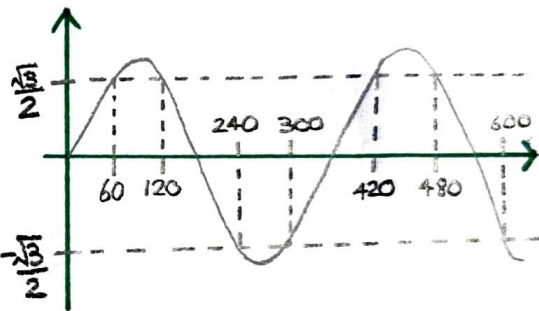


FIGURE 1

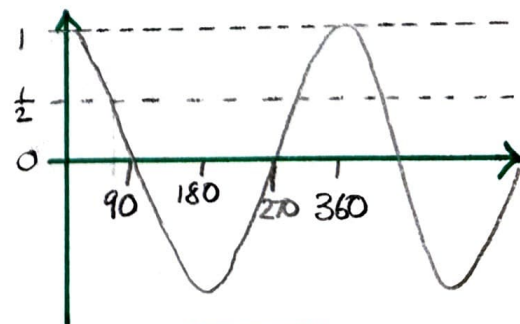


FIGURE 2