2 The curve $y = x^3 - 6x + 3$ has turning points at $x = \alpha$ and $x = \beta$, where $\beta > \alpha$.

Find

$$\int_{\alpha}^{\beta} x^3 - 6x + 3 \, dx$$

A $-8\sqrt{2}$

B -10

C $-10 + 6\sqrt{2}$

D 0

E $12 - 8\sqrt{2}$

F $6\sqrt{2}$

G 12

First let's find α and β by differentiating

$$\frac{dy}{dx} = 3x^2 - 6$$

Setting this equal to zero gives

$$3x^2 - 6 = 0$$

$$x = \pm\sqrt{2}$$

note: I know which one is which because $\beta > \alpha$

so $\alpha = -\sqrt{2}$ and $\beta = \sqrt{2}$ and I have

$$\int_{-\sqrt{2}}^{\sqrt{2}} x^3 - 6x + 3 \, dx$$

$$= \left[\frac{x^4}{4} - 3x^2 + 3x\right]_{-\sqrt{2}}^{\sqrt{2}}$$

$$= \left(\frac{(\sqrt{2})^4}{4} - 3(\sqrt{2})^2 + 3\sqrt{2}\right) - \left(\frac{(-\sqrt{2})^4}{4} - 3(-\sqrt{2})^2 - 3\sqrt{2}\right)$$

$$= 3\sqrt{2} + 3\sqrt{2}$$

$$= 6\sqrt{2}$$

which is option F.