TMUA 2021 Paper 1 Q3

An arithmetic progression and a convergent geometric progression each have first term $\frac{1}{2}$

another word for 'sequence'

* The sum of the second terms of the two progressions is $\frac{1}{2}$

** The sum of the third terms of the two progressions is $\frac{1}{8}$

What is the sum to infinity of the geometric progression?

A -2

B -1

C $-\frac{1}{2}$

D $-\frac{1}{3}$

E $\frac{1}{3}$

F $\frac{1}{2}$

G 1

H 2

this is equivalent to $-1<r<1$

The sum to infinity, S_∞, of a convergent geometric progression is $S_\infty = \frac{a}{1-r}$, with $|r|<1$ where a is the first term and r is the common ratio for the geometric progression.
Let d be the common difference for the arithmetic progression.

	1st term	2nd term	3rd term
arithmetic progression	$\frac{1}{2}$	$\frac{1}{2}+d$	$\frac{1}{2}+2d$
geometric progression	$\frac{1}{2}$	$\frac{r}{2}$	$\frac{r^2}{2}$

TABLE 1

TABLE 1 compiles some information about the terms of the two progressions. Using statements * and **, I can form equations which can be solved simultaneously to find r.

From *, I have $\frac{1}{2}+d+\frac{r}{2}=\frac{1}{2}$

$\Leftrightarrow \quad d=-\frac{r}{2}$

From **, I have $\frac{1}{2}+2d+\frac{r^2}{2}=\frac{1}{8}$

substituting $d=-\frac{r}{2}$ and multiplying all across by 8 gives, $4-8r+4r^2=1$

$\Leftrightarrow \; 4r^2-8r+3=0$

$\Leftrightarrow \; (2r-3)(2r-1)=0$

I need $|r|<1$, so the only valid solution from this equation is $r=-\frac{1}{2}$

Using this, along with $a=\frac{1}{2}$ to evaluate S_∞ I get

$$S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

so the correct answer is G