

TMUA 2021 Paper 1 Q5

The function f is such that

$$f(mn) = \begin{cases} f(m)f(n) & \text{if } mn \text{ is a multiple of 3} \\ mn & \text{if } mn \text{ is not a multiple of 3} \end{cases}$$

for all positive integers m and n .

Given that $f(9) + f(16) - f(24) = 0$, what is the value of $f(3)$?

A $\frac{8}{3}$

B $2\sqrt{2}$

C 3

D $\frac{16}{5}$

E $3\sqrt{2}$

F 4

If we try to evaluate $f(3)$ using the function rules, we won't get anywhere useful:

$$f(3) = f(3 \times 1) = f(3)f(1) = f(3) \times 1 = f(3)$$

So we use the given equation, taking care to notice any powers of 3 when factorising the input

$$f(9) + f(16) - f(24) = 0$$

$$f(3 \times 3) + 16 - f(8 \times 3) = 0$$

$$f(3)f(3) + 16 - f(8)f(3) = 0$$

$$[f(3)]^2 + 16 - 8f(3) = 0$$

note: 16 is not a multiple of 3

note: 8 is not a multiple of 3

This is a quadratic in $f(3)$ which we can factorise and solve to find the value of $f(3)$

$$[f(3)]^2 - 8f(3) + 16 = 0$$

$$(f(3) - 4)(f(3) - 4) = 0$$

$$(f(3) - 4)^2 = 0$$

which gives a repeated solution of

$$f(3) = 4$$

so the correct answer is option F