

TMUA 2021 Paper 1 Q6

The function f is given by

$$f(x) = \frac{\cos x + 3}{7 + 5 \cos x - \sin^2 x}$$

Find the positive difference between the maximum and the minimum values of $f(x)$.

- A 0
B $\frac{1}{3}$
C $\frac{1}{2}$

We first note that this is a function in $\cos x$ and $\sin x$ but we can eliminate the $\sin^2 x$ using the identity $\sin^2 x + \cos^2 x = 1$

- D $\frac{2}{3}$

$$f(x) = \frac{\cos x + 3}{7 + 5 \cos x - (1 - \cos^2 x)}$$

- E 1

- F 2

$$f(x) = \frac{\cos x + 3}{\cos^2 x + 5 \cos x + 6}$$

and the denominator factorises to give

$$f(x) = \frac{\cos x + 3}{(\cos x + 2)(\cos x + 3)}$$

now $-1 \leq \cos x \leq 1$, so we have $\cos x + 2 > 0$
and $\cos x + 3 > 0$ so we can divide numerator
and denominator by $(\cos x + 3)$ to give

$$f(x) = \frac{1}{\cos x + 2}$$

This will take its minimum value when $\cos x = 1$ and its maximum value when

$$\cos x = -1$$

$$\text{minimum value} = \frac{1}{1+2} = \frac{1}{3}$$

$$\text{maximum value} = \frac{1}{-1+2} = 1$$

$$\text{and } 1 - \frac{1}{3} = \frac{2}{3}$$

so the correct answer is option D