# TMUA 2021 Paper 1 Q7

The function f is such that $f(0) = 0$, and $xf(x) > 0$ for all non-zero values of $x$.

It is given that

$$\int_{-2}^{2} f(x)\,dx = 4$$

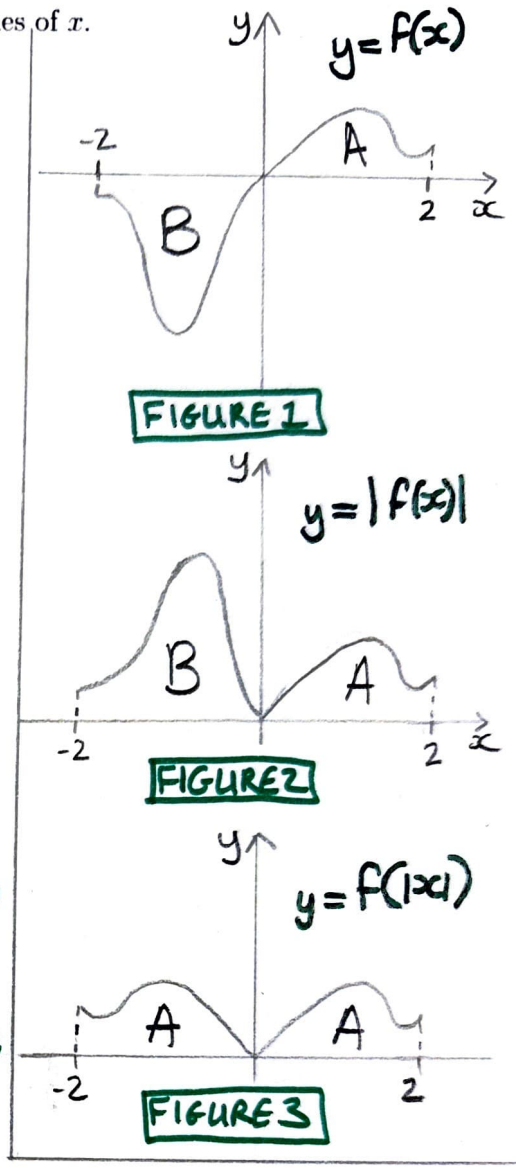
and

$$\int_{-2}^{2} |f(x)|\,dx = 8$$

Evaluate

$$\int_{-2}^{0} f(|x|)\,dx$$

A −8
B −6
C −4
D −2
E 2
F 4
G 6
H 8




f(0)=0 tells us that f(x) passes through the origin.

$xf(x) > 0$ tells us that when $x>0$, we must have $f(x)>0$ and when $x<0$, we must have $f(x)<0$

FIGURE 1 shows a function which satisfies these criteria, i.e. one such f(x), between $x=-2$ and $x=2$.

FIGURE 2 shows $|f(x)|$ and FIGURE 3 shows $f(|x|)$

Let A and B be the values of the areas between f(x) and the x-axis, as labelled in figure 1

From FIGURE 3 I can see that $\int_{-2}^{0} f(|x|)\,dx = A$ and this is the value I need.

From FIGURE 1, and the first result given, I have $A - B = \int_{-2}^{2} f(x)\,dx = 4$

i.e. $A - B = 4$

From FIGURE 2, and the next result given, I have $A + B = \int_{-2}^{2} |f(x)|\,dx = 8$

i.e. $A + B = 8$

Solving simultaneously I have

$A + B = 8$
$A + (A - 4) = 8$
$2A = 12$
$A = 6$

so the correct answer is G