TMUA 2021 Paper 1 Question 8

The line $y = 2x + 3$ meets the curve $y = x^2 + bx + c$ at exactly one point.

The line $y = 4x - 2$ also meets the curve $y = x^2 + bx + c$ at exactly one point.

What is the value of $b - c$?

A -9

B -5.5

C -1

D 5

E 6

F 14

For the line $y=2x+3$ to meet the curve $y=x^2+bx+c$, I must have $2x+3=x^2+bx+c$, at that point. The same applies for the line $y=4x-2$, and so I have the following

$$x^2+bx+c=2x+3 \quad \text{and} \quad x^2+bx+c=4x-2$$

these equations can be rearranged as follows

$$x^2+(b-2)x+(c-3)=0 \quad \text{and} \quad x^2+(b-4)x+(c+2)=0$$

In order for the lines to meet the curve at exactly one point, I need the respective discriminants to have a value of zero

$$(b-2)^2-4(c-3)=0 \quad \text{and} \quad (b-4)^2-4(c+2)=0$$

① $b^2-4b-4c+16=0$ ② $b^2-8b-4c+8=0$

I now have 2 equations with 2 unknowns.
Subtracting ② from ① gives

$$4b+8=0$$
$$4b=-8$$
$$b=-2$$

and substituting this value of b into ① gives

$$(-2)^2-4(-2)-4c+16=0$$
$$-4c=-28$$
$$c=7$$

and finally, $b-c=-2-7=9$

so the correct answer is A