TMUA 2021 Paper 1 Question 9

Find the area enclosed by the graph of

$$|x| + |y| = 1$$

A $\frac{1}{2}$

B 1

C 2

D 4

E $\frac{1}{2}\sqrt{2}$

F $\sqrt{2}$

G $2\sqrt{2}$

We can think about the modulus function, $|x|$, as having two cases, i.e. $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Considering each quadrant of the xy-plane, helps us to see that we will need to consider 4 separate cases.

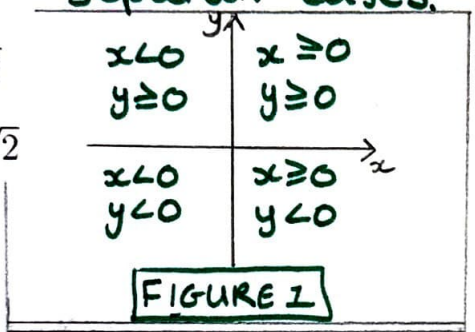

FIGURE 1

FIGURE 1 shows whether x and y are positive or negative in each quadrant.

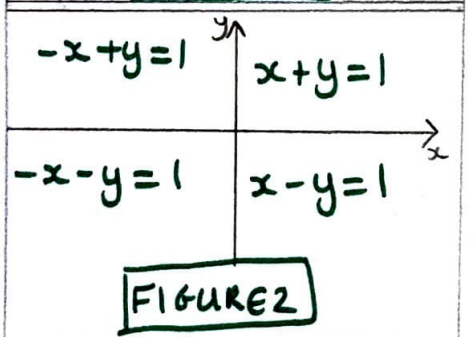

FIGURE 2

FIGURE 2 shows the linear equation we need to graph in each quadrant, which will make up the graph of $|x|+|y|=1$

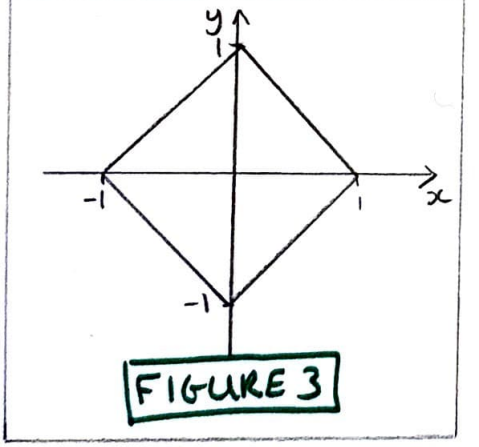

FIGURE 3

FIGURE 3 shows the full graph of $|x|+|y|=1$

To find the area enclosed we can either:

① use pythagoras' theorem to find the side length of $\sqrt{2}$ and the area follows as $(\sqrt{2})^2 = 2$

② Spot that each quadrant has half a unit square which gives a total area of $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2} = 2$

so the correct answer is C