TMUA 2021 Paper 2 Question 11

A student attempts to solve the following problem, where a and b are non-zero real numbers:

{First part of the statement}

{second part of the statement}

Show that **if** $a^2 - 4b^3 \geq 0$ **then** there exist real numbers x and y such that $a = xy(x + y)$ and $b = xy$.

Consider the following attempt:

$$(x - y)^2 \geq 0 \qquad \text{(I)}$$

{This is true for any value of x and y}

$$\text{so} \quad x^2 + y^2 - 2xy \geq 0 \qquad \text{(II)}$$

{This line follows by expanding $(x-y)^2$}

$$\text{so} \quad (x + y)^2 - 4xy \geq 0 \qquad \text{(III)}$$

{This line is effectively the same as the previous one and we can check this by expanding $(x+y)^2$}

$$\text{so} \quad x^2y^2(x + y)^2 - 4x^3y^3 \geq 0 \qquad \text{(IV)}$$

{This line follows by multiplying both sides of the inequality by x^2y^2, which can be done because $x^2y^2 \geq 0$}

$$\text{so} \quad a^2 - 4b^3 \geq 0 \qquad \text{(V)}$$

{To get to this line the student has assumed $a = xy(x+y)$ and $b = xy$, in order to arrive at $a^2 - 4b^3 \geq 0$}

Which of the following best describes this attempt?

A It is completely correct.

B It is incorrect, but it would be correct if written in the reverse order.

C It is incorrect, but the student has correctly proved the converse.

D It is incorrect because there is an error in line (II).

E It is incorrect because there is an error in line (III).

F It is incorrect because there is an error in line (IV).

Everything up to and including line (IV) is true for all values of x and y and doesn't assume anything.

To get to line (V) the student assumes the second part of the statement and arrives at the first part of the statement.
Therefore the student has correctly proved the converse

and the correct answer is C.