TMUA 2021 Paper 2 Question 12

Which of the following statements about polynomials f and g is/are true?

I **If** $f(x) \geq g(x)$ for all $x \geq 0$, **then** $\int_0^x f(t)\,dt \geq \int_0^x g(t)\,dt$ for all $x \geq 0$.

II **If** $f(x) \geq g(x)$ for all $x \geq 0$, **then** $f'(x) \geq g'(x)$ for all $x \geq 0$.

This means the gradient of $f(x)$ is greater than or equal to the gradient of $g(x)$

III **If** $f'(x) \geq g'(x)$ for all $x \geq 0$, **then** $f(x) \geq g(x)$ for all $x \geq 0$.

This means $f(x)$ lies above or in line with $g(x)$ on the x-y plane.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

In statement I $f(x)$ must lie above or in line with $g(x)$, in this case the area between $f(x)$ and the x-axis would always be greater than or equal to the area between $g(x)$ and the x-axis. FIGURE 1 shows an example where both $f(x)$ and $g(x)$ are positive and figure 2 shows an example where both $f(x)$ and $g(x)$ are negative.

In each case $\int_0^x f(t)\,dt$ equals the value of the area shaded +

and $\int_0^x g(t)\,dt$ equals the value of the area shaded +

Therefore statement I is true

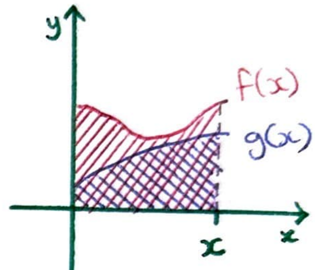

FIGURE 1

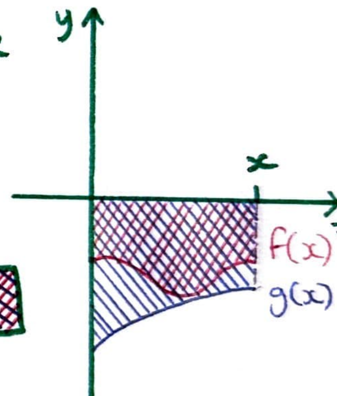

FIGURE 2

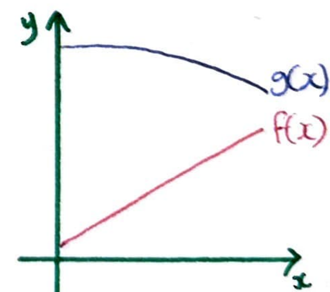

FIGURE 3

To investigate statement II, consider that FIGURES 1 and 2 both satisfy $f(x) \geq g(x)$ but in each FIGURE $f'(x) < g'(x)$ for at least the first half of the sketched functions. Therefore statement II is not true.

To investigate statement III, consider FIGURE 3 in which $f'(x) > g'(x)$. For the sections of $f(x)$ and $g(x)$ that have been sketched, $g(x)$ lies above $f(x)$, i.e. $g(x) > f(x)$. This provides a counter example to statement III. Therefore satement III is not true

so the correct answer is B