TMUA 2021 Paper 2 Question 14

Consider the following simultaneous equations, where p is a real number:

$$\textcircled{1} \quad p2^x + \log_2 y = 2$$

$$\textcircled{2} \quad 2^x + \log_2 y = 1$$

What is the complete range of p for which these simultaneous equations have a real solution (x, y)?

A $p < 1$

B $p \neq 1$

C $p > 1$

D $p < 1$ or $p > 2$

E $p \neq 1$ and $p < 2$

F $p > 1$ and $p < 2$

G $p > 2$

H All real values of p

Lets consider ① − ② and we would get

$$(p-1)2^x = 1$$

$$2^x = \frac{1}{p-1}$$

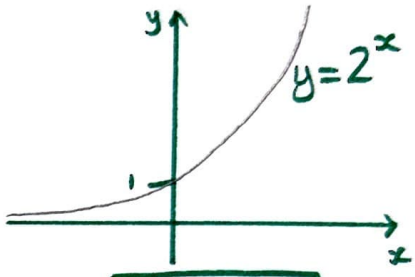

FIGURE 1

This step is valid because in order for ① and ② to hold simultaneously, $p \neq 1$ (to verify this consider what happens when substituting $p=1$ into ① and compare with ②)

FIGURE 1 shows the graph of $y = 2^x$ which is always positive, therefore, we need $p > 1$, in order to guarantee that x is a real number

Now let's consider what we need in order for y to be a real number. Substituting $2^x = \frac{1}{p-1}$ into ② gives

$$\frac{1}{p-1} + \log_2 y = 1$$

$$\log_2 y = 1 - \frac{1}{p-1}$$

note: we already know $p \neq 1$

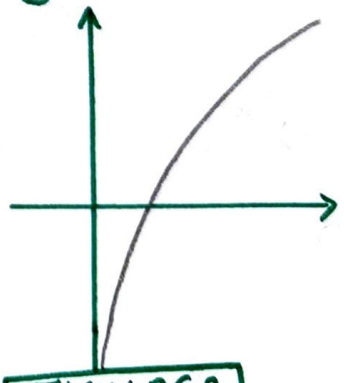
FIGURE 2

Now $\log_2 y$ can take any value (see figure 2 for reference) so we need no further restriction on p in order that a real value of y exists.

Therefore the correct answer is C