

# TMUA 2021 Paper 2 Question 15



A circle has equation

$$x^2 + ax + y^2 + by + c = 0$$

where  $a$ ,  $b$  and  $c$  are non-zero real constants.

Which one of the following is a necessary and sufficient condition for the circle to be tangent to the  $y$ -axis?

A  $a^2 = 4c$

B  $b^2 = 4c$

C  $\frac{a}{2} = \sqrt{\frac{a^2 + b^2}{4}} - c$

D  $\frac{b}{2} = \sqrt{\frac{a^2 + b^2}{4}} - c$

E  $-\frac{a}{2} = \sqrt{\frac{a^2 + b^2}{4}} - c$

F  $-\frac{b}{2} = \sqrt{\frac{a^2 + b^2}{4}} - c$

Let the point  $(0, y)$  be a generic point on the  $y$ -axis.

For  $(0, y)$  to lie on the circle we must have

$$0^2 + a(0) + y^2 + by + c = 0$$

that is  $y^2 + by + c = 0$

The discriminant of this quadratic is  $b^2 - 4c$

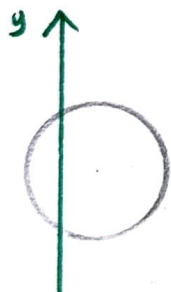
If  $b^2 - 4c > 0$ , then we would have scenario 1

If  $b^2 - 4c < 0$ , then we would have scenario 2

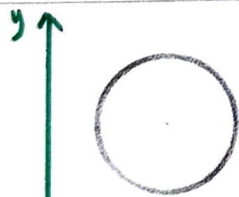
If  $b^2 - 4c = 0$ , then we would have scenario 3 in which the circle is tangent to the  $y$ -axis

Therefore  $b^2 - 4c = 0$  is the necessary and sufficient condition we need

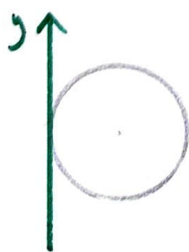
$b^2 - 4c = 0 \iff b^2 = 4c$   
so the correct answer is **B**.



SCENARIO 1



SCENARIO 2



SCENARIO 3