

TMUA 2021 Paper 2 Question 18



A student chooses two distinct real numbers x and y with $0 < x < y < 1$.

The student then attempts to draw a triangle ABC with:

$$AB = 1$$

$$\sin A = x$$

$$\sin B = y$$

Which of the following statements is/are correct?

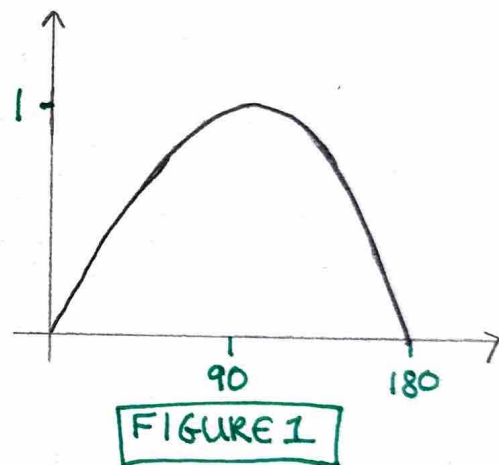
- I For some choice of x and y , there is exactly **one** triangle the student could draw.
- II For some choice of x and y , there are exactly **two** different triangles the student could draw.
- III For some choice of x and y , there are exactly **three** different triangles the student could draw.

(Note that congruent triangles are considered to be the same.)

- A none of them
- B I only
- C II only
- D III only
- E I and II only
- F I and III only
- G II and III only
- H I, II and III

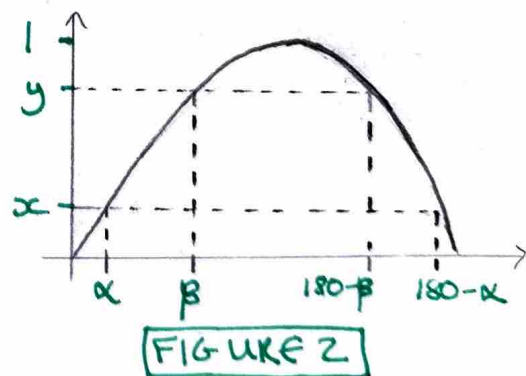
FIGURE 1 shows the graph of the sine function for inputs between 0° and 180°

note: any angle in a triangle must be between these values



We start with choosing an x and a y that satisfy $0 < x < y < 1$

FIGURE 2 shows one such choice. This leads to two possibilities for A , labelled α and $180-\alpha$, and two possibilities for B , labelled β and $180-\beta$ in FIGURE 2



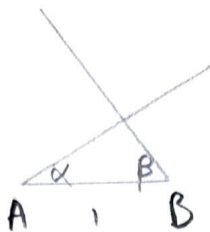
Let AB be the base of the triangle, which we are told has length 1 unit.

This leads to 4 possible cases, which we investigate in turn.

Case 1

$$A = \alpha$$

$$B = \beta$$

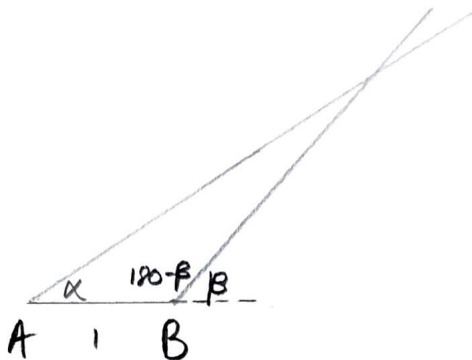


In this case, with A and B being acute, this would certainly lead to a triangle.

Case 2

$$A = \alpha$$

$$B = 180 - \beta$$

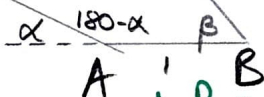


In this case, A is acute and B is obtuse, and because $\beta > \alpha$, this would lead to a triangle.

Case 3

$$A = 180 - \alpha$$

$$B = \beta$$

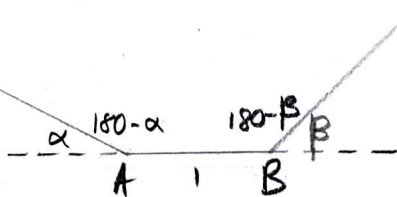


In this case, A is obtuse and B is acute, but because $\beta > \alpha$, this would not lead to a triangle.

Case 4

$$A = 180 - \alpha$$

$$B = 180 - \beta$$



In this case, with both A and B being obtuse, this would not lead to a triangle.

In conclusion, for some choice of x and y , there are exactly 2 different triangles the student could draw, which shows statement II only is correct

so the correct answer is option C.