

TMUA 2021 Paper 2 Question 3



A student is chosen at random from a class. Each student is equally likely to be chosen.

Which of the following conditions is/are **necessary** for the probability that the student wears glasses to equal $\frac{4}{15}$?

- I Exactly 11 students in the class do not wear glasses.
- II The number of students in the class is divisible by 3.
- III The class contains 30 students, and 8 of them wear glasses.

- A none of them
- B I only
- C II only
- D III only
- E I and II only
- F I and III only
- G II and III only
- H I, II and III

Note: for statements A and B

- To check if A is sufficient for B, we start with A and investigate whether it guarantees B or not i.e. $A \Rightarrow B$ can be read "A implies B" or "A is sufficient for B"
- To check if A is necessary for B, we start with B and investigate whether it guarantees A or not i.e. $A \Leftarrow B$ can be read "A is implied by B" or "A is necessary for B"

Let $P(g)$ be the probability that the student wears glasses.

① Investigating $I \Leftarrow P(g) = \frac{4}{15}$

Starting with $P(g) = \frac{4}{15}$ we could have a class of 15 students, 11 of which do not wear glasses but we could also have a class of 30 students, 22 of which do not wear glasses because $P(g) = \frac{4}{15} = \frac{8}{30} = \dots$ so $P(g) = \frac{4}{15}$ does not guarantee statement I. Therefore I is not necessary for $P(g) = \frac{4}{15}$.

② Investigating $II \Leftarrow P(g) = \frac{4}{15}$

Starting with $P(g) = \frac{4}{15}$ we can see the denominator of $P(g)$ is a multiple of 3, therefore the number of students in the class must be a multiple of 3, so II is necessary for $P(g) = \frac{4}{15}$.

③ Investigating $III \Leftarrow P(g) = \frac{4}{15}$

Because $P(g) = \frac{4}{15} = \frac{8}{30}$ we could have a class of 30 students, 8 of which wear glasses, but I also have $P(g) = \frac{4}{15} = \frac{12}{45} = \dots$ which could give a class of 45 students, 12 of whom wear glasses so there are many possible scenarios and III is not necessary for $P(g) = \frac{4}{15}$ so the correct answer is C.