



### TMUA 2021 Paper 2 Question 7

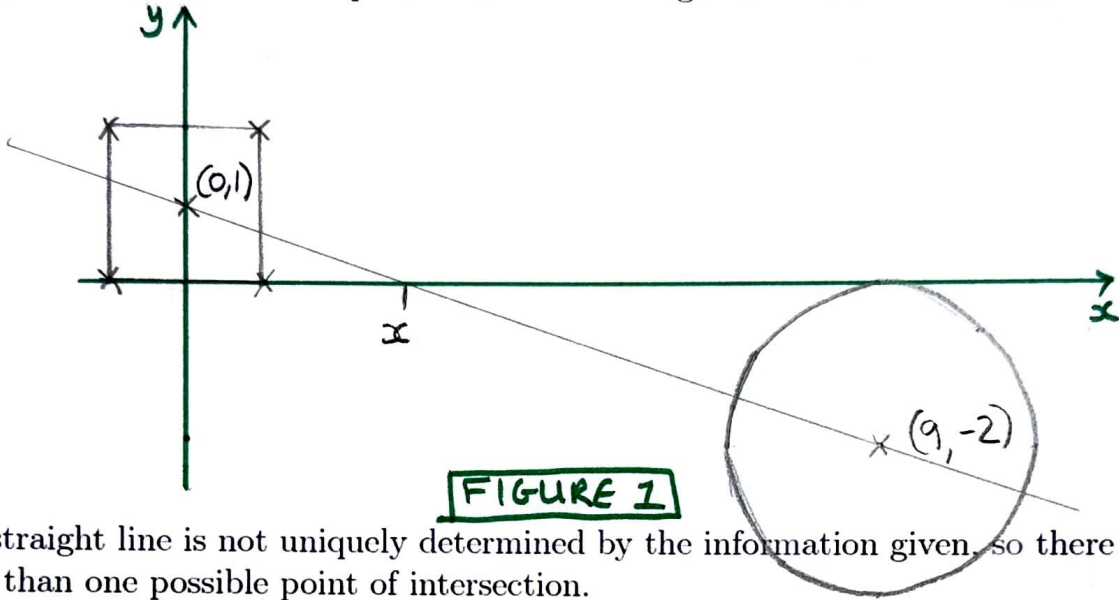
A circle has equation  $(x - 9)^2 + (y + 2)^2 = 4$

A square has vertices at  $(1, 0)$ ,  $(1, 2)$ ,  $(-1, 2)$  and  $(-1, 0)$ .

A straight line bisects both the area of the circle and the area of the square.

What is the  $x$ -coordinate of the point where this straight line meets the  $x$ -axis?

- A 2
- B 3
- C 4
- D 4.5
- E 5
- F 6



**FIGURE 1**

G The straight line is not uniquely determined by the information given, so there is more than one possible point of intersection.

H There is no straight line that bisects both the area of the circle and the area of the square.

Let  $l$  be the line that bisects the circle and the square. Let  $x$  be the  $x$ -coordinate of the point where  $l$  meets the  $x$ -axis.

$l$  must pass through the centre of the circle and through the centre of the square.

We can see that the circle has centre  $(9, -2)$  from the circle equation.

We can find the centre of the square by taking the midpoint between either pair of diagonally opposite vertices e.g. the midpoint of  $(1, 0)$  and  $(-1, 2)$  is  $(\frac{1-1}{2}, \frac{0+2}{2}) = (0, 1)$

Let  $m$  be the gradient of  $l$ , then  $m = \frac{-2-1}{9-0} = -\frac{1}{3}$

substituting  $(x_1, y_1) = (0, 1)$ ,  $m = -\frac{1}{3}$  and  $y = 0$  into  $y - y_1 = m(x - x_1)$  gives

$$y - y_1 = m(x - x_1)$$

$$0 - 1 = -\frac{1}{3}(x - 0)$$

$$-1 = -\frac{1}{3}x$$

$$x = 3$$

so the correct answer is **B**

We need this because we are looking for the value  $x$  in the point  $(x, 0)$  where  $l$  meets the  $x$ -axis. In fact,  $y = 0$  is the equation of the  $x$ -axis!