Lecture 12

Flow Past an Aerofoil

12.1 Conformal Mapping

Recall, last lecture we used a complex potential to describe the flow around a cylinder. The idea in this lecture is to use a conformal transformation $Z = f(z)$ to map this circle into an aerofoil like object, figure 12.1. After such a transformation the flow past the

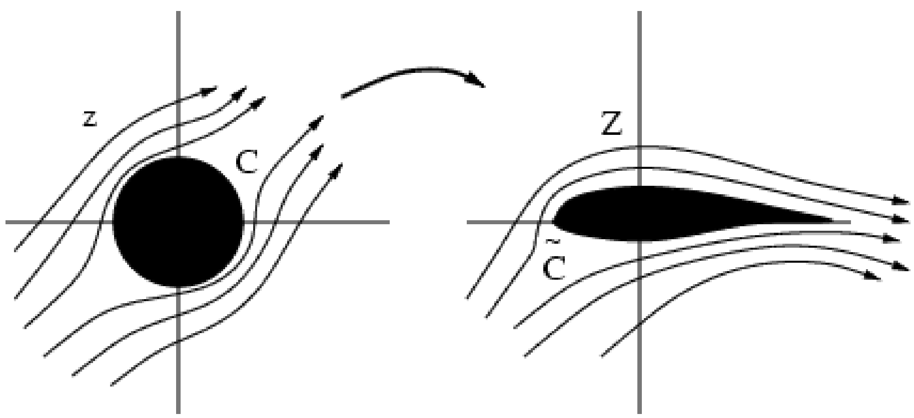

Figure 12.1: Conformal Transformation

aerofoil (on plane $Z = (X, Y)$) will be given by the complex potential

$$\mathbb{X}(Z) = \chi(F(z)),$$

where F is the inverse of function f and χ is the original complex potential for a flow past a cylinder.

Note 1 - $\mathbb{X}(Z)$ is an analytic function of Z, i.e. both the real Φ and imaginary Ψ parts of it satisfy *Laplace's Equation.* They describe the potential and streamfunction of our 2D irrotational flow.

Note 2 - The value of $\mathbb{X}$ on the surface of the aerofoil is the same as that of χ on the cylinder (circle). The same is true for the streamfunctions, i.e. $\Psi = const.$ That is, the surface of the aerofoil is a streamline and the *new flow* satisfies the right BC, $u_n = 0$ on the aerofoil. One should also to note that Φ and ϕ are the same on the boundary $\Rightarrow$ the circulation is the *same* because Γ is the change in Φ after one circuit around the boundary.

12.2 Joukowski Transformation

We will now make use of the Joukowski Transformation to map our cylinder (circle) to a symmetric aerofoil. That is,

$$Z = z + \frac{a^2}{z},$$

We will start by taking the complex potential $\chi(z)$ for a flow past a cylinder. Shifting our cylinder to the left by λ, figure 12.2, and rotating it by angle α, we find,

$$\chi = U\left[(z+\lambda)e^{-i\alpha} + \frac{(a+\lambda)^2}{(z+\lambda)}e^{i\alpha}\right] - \frac{i\Gamma}{2\pi}\ln(z+\lambda).$$

We know that $\partial_Z \mathbb{X} = U + iV$ (where U and V are the components of the velocity), so using the chain rule we find

$$\partial_Z \mathbb{X} = \frac{\partial_z \chi}{dZ/dz} = \left\{U\left[e^{-i\alpha} - \left(\frac{a+\lambda}{z+\lambda}\right)^2 e^{i\alpha}\right] - \frac{i\Gamma}{2\pi(z+\lambda)}\right\} \Big/ \left(1 - \frac{a^2}{z^2}\right),$$

where we have used the Joukowski Transformation to determine dZ/dz. We see that there is only one value of Γ which makes the numerator 0 at $z = a$, that is

$$\Gamma = -4\pi U(a+\lambda)\sin\alpha. \tag{12.1}$$

For all other Γ's the velocity is ∞ at $z = a$ (because of a 0 denominator). Infinite velocity is unphysical. In real life we find that nature selects the Γ corresponding to $\mathbf{U}$ being finite everywhere, i.e. the Γ given by (12.1). This condition is known as the *Kutta-Joukowski condition.*

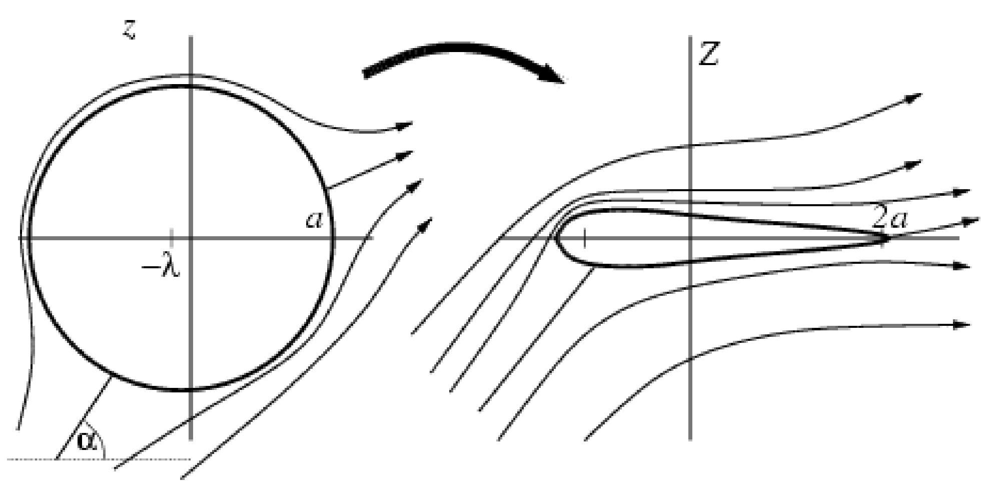

Figure 12.2: Flow Past a Symmetric Aerofoil