Lecture 8

Inviscid and Viscous Fluids

8.1 Intermediate Summary

Although we are only about half way through our study of inviscid solutions, it is useful to stop at this point and briefly consider properties of viscous fluids in order to understand when and where the inviscid solutions provide a good approximation to the dynamics of a real fluid (which is *always* viscous).

Consider the NSE (we will derive this later),

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u}. \tag{8.1}$$

The last term describes viscosity, one can estimate it as,

$$\nu \frac{U}{L^2},$$

where U and L are the characteristic velocity and lengthscale (of the velocity variation), respectively. To understand how important this term is, one can compare it to the second term on the LHS (the convective or advection term), which is,

$$\sim \frac{U^2}{L}.$$

The ratio of the convective term to the viscosity term is then approximately,

$$\frac{U^2}{L} \Big/ \frac{\nu U}{L^2} = \frac{UL}{\nu}.$$

This combination is called the *Reynolds Number Re*,

$$Re = \frac{UL}{\nu}. \tag{8.2}$$

Thus, the viscosity can be ignored when $\boxed{Re \gg 1}$. Consider a flow about a body of characteristic size a, figure 8.1.

Naive Intuition tells us that the body will deflect the streamlines to a distance of order

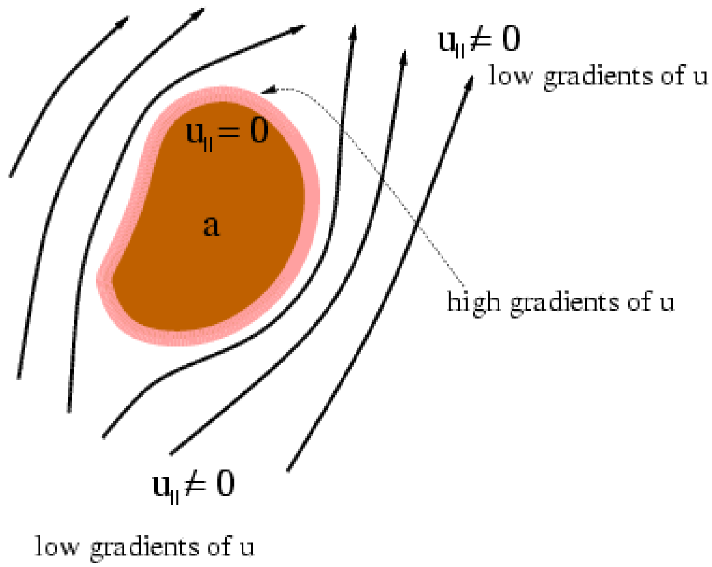

Figure 8.1: Flow Around a Body with a BL

a from its initial position and, therefore, one can use a as a characteristic lengthscale, i.e. use $l \sim a$ in evaluating Re. Then, if

$$Re = \frac{Ua}{\nu} \gg 1,$$

we can neglect viscosity.

In reality, this naive argument is valid only in part of the volume occupied by the fluid when $Re \gg 1$, which is mathematically related to the following facts.

Fact 1: In general, no inviscid solution can accommodate the no-slip BC's relevant to the real (viscous) fluids. "*No-slip*" means that the tangential component of velocity $u_{||} = 0$ at the fixed boundary. This is because the Euler equation is a lower order PDE than the NSE, and "no-slip" over-defines the problem. As a result, a thin layer is at least needed (near the boundary), with high velocity gradients (this makes the viscosity term large), to *slow down* the fluid flow from the velocity of the inviscid solution (outside of the layer) to zero (at the boundary). Such a layer is called the *Boundary Layer* (BL), figure 8.1.

Fact 2: In fact, it is not clear *apriori* that the BL must remain small. This would be the case if all the streamlines remain "attached" to the boundary, i.e. no streamline originating in the boundary layer penetrates deeply into the fluid. Under some conditions nature chooses solutions which have streamlines that separate from the boundary, figure 8.2. Vorticity generated in the thin boundary layer propagates into the bulk of the fluid, and the fluid motion becomes turbulent there.

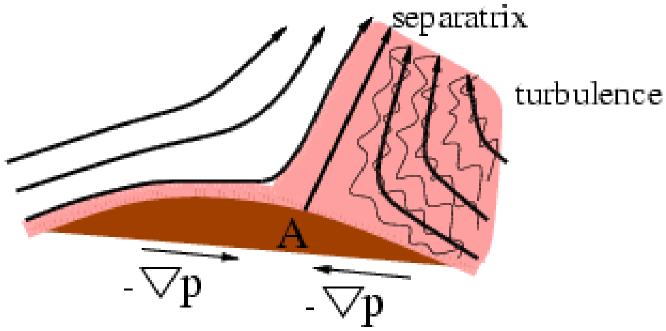

Figure 8.2: Separation

8.1.1 Factors Affecting Separation

External

The external factors affecting flow separation (with respect to the BL) are,

- Adverse Pressure Gradients, figure 8.2. The pressure forces $-\nabla p$ are against the inpinging flow, downstream of point A. They squeeze the fluid elements and push the fluid away from the surface.
- Sharp Corners, figures 8.3, 8.4

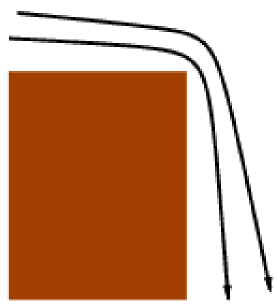

Figure 8.3: Sharp Corners: an inviscid solution

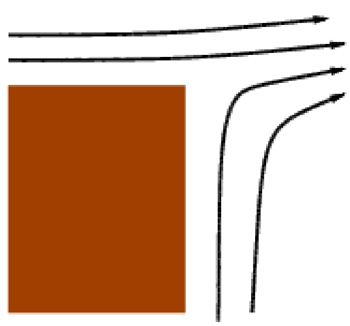

Figure 8.4: Sharp Corners: an alternative inviscid solution

Both these factors help in separation and we will consider them in more detail later on. In real life, both these solutions exist, however nature tends to select a flow which looks more like that found in figure 8.5.

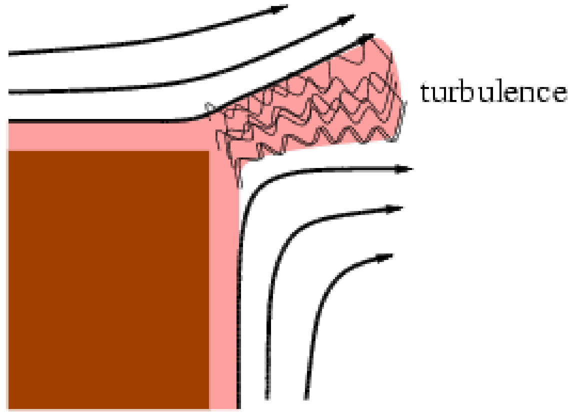

Figure 8.5: Turbulence

Internal

Internal factors also affect flow separation. In fact turbulence plays a very important role. Consider, a flow past a circular cylinder. For a Reynolds number less than 20000 we have a laminar BL, figure 8.6 However, as the Reynold's number increases the BL itself becomes

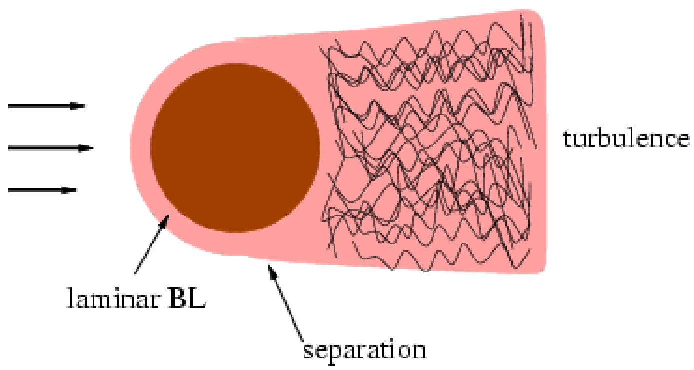

Figure 8.6: Laminar BL with Separation. $Re < 20000$

turbulent, figure 8.7. One should notice that added turbulence suppresses separation. This result is also known as the "*Drag Crisis*", figure 8.8. That is, the separation causes a drag. When there is no separation there is no drag - this is D'Alembert's Paradox.

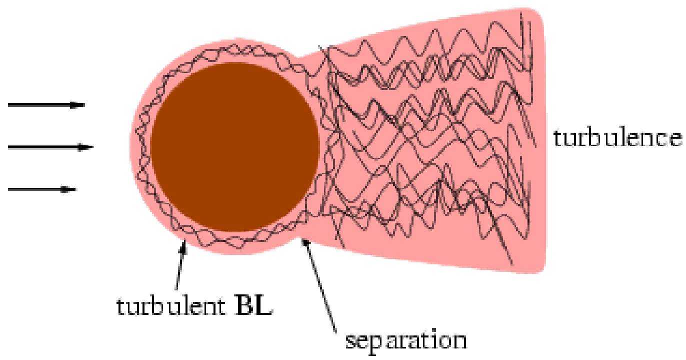

Figure 8.7: Turbulent BL. $Re > 20000$

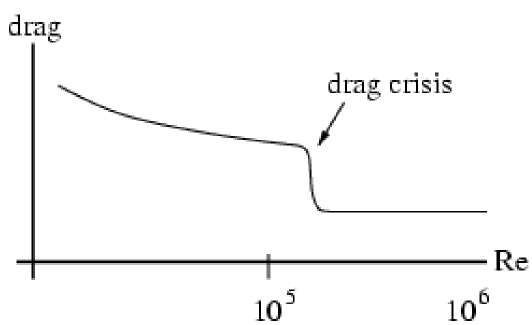

Figure 8.8: The Drag Crisis