Lab 5

Questions with a (*) next to them form part of homework 2. Due in 2/11/2015. [Q 5]

1. Consider a random sample $X_1, \ldots, X_n \sim$Uniform(θ_1, θ_2), $\theta_1 < \theta_2$ unknown.
   (a) Find (non-trivial) sufficient statistics for (θ_1, θ_2).
   (b) Find MLE $(\hat{\theta}_1, \hat{\theta}_2)$ for (θ_1, θ_2).
   (c) As a function of (θ_1, θ_2) and n, what is the probability that $|\hat{\theta}_1 - \theta_1| \leq 1$?
2. Calculate the log-likelihood for a iidrv random sample from the normal $N(\mu, \sigma^2)$ distribution. Calculate MLE for μ and σ^2.
3. Use Lagrange multipliers to find the MLE for the Multinomial distribution.
4. Newton-Raphson method: Use the following code to find $\sqrt{3}$ and $\sqrt[3]{2}$.

```
newton.raphson <- function(f,df,x=0,reps=100) {
  a=c(x)   for (i in 1:reps) {
    x=x-f(x)/df(x)
    a=c(a,x)
  }
  print(a)
  plot(a,type="l")
}
# Create functions f and df as appropriate and then
# call newton.raphson(f,df,start.point,number.of.reps)
```

5. (*) EM-algorithm: Random samples $X_1, \ldots, X_n$ are drawn (independently) from the $N(\mu_1, 1)$ distribution with probability 0.5, and are drawn from the $N(\mu_2, 2)$ distribution with probability 0.5, i.e.

$$f(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-\mu_1)^2}{2}\right) + \frac{1}{2} \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-\mu_2)^2}{2}\right)$$

   Find data at `http://www2.warwick.ac.uk/fac/sci/mathsys/courses/msc/ma930/resources`

   (a) Use the hard EM algorithm to find MLE for μ_1 and μ_2.
   (b) Use the EM algorithm to find MLE for $\mu_1$and μ_2.
6. Find the estimator/statistic $\delta(X)$ that minimizes the posterior expected loss

$$\mathbb{E}_{\theta|X}(L(\theta, \delta(X)) = \int_\theta L(\theta, \delta(X)) f(\theta \mid X) d\theta$$

   given posterior distribution $f(\theta \mid x)$ when the loss function is

   (a) quadratic loss $L(\theta, \delta) = (\theta - \delta)^2$,
   (b) absolute value loss $L(\theta, \delta) = |\theta - \delta|$.
7. Let $X_1, \ldots, X_n$ denote a random sample from the exponential distribution:

$$f(x_1 \mid \theta) = \theta \exp(-\theta x), \qquad \theta, x > 0$$

   Calculate the MLE $\hat{\theta}$ for θ, and the bias, variance and mean squared error. Use the mean squared error to show that $\hat{\theta}$ is consistant.