Exam topics MA933

Basic probability, DTMCs

- Definition of probability space, events, probability distribution, independence, conditional probability, law of total probability
- Random variables, distribution function (CDF), mass function, PDF, expectation, variance, joint distribution, marginal (definitions and compute simple examples)
- Simple random walk Y_n, compute expectation and variance of Y_n, compute distribution
- State weak law of large numbers and central limit theorem!
- Definition of discrete-time Markov process, sample path, simple examples (including non-Markov processes)
- Transition functions and Chapman Kolmogorov equations (including proof), matrix formulation, computation of probabilities for simple examples
- Stationary/reversible distributions, detailed balance, eigenvectors of transition matrix
- expansion of <pi_n | in terms of powers of eigenvalues, state Gershgorin theorem and Perron-Frobenius theorem
- absorbing states, linear recursion for absorption probabilities, solution of those
- Random walks with different boundary conditions: transition matrices, stationary distributions
- Explain Toom's model and sketch typical patterns (Q1.4)

Continuous-time Markov chains (CTMCs)

- Definition and path space of CTMCs, sample paths
- Transition functions and Chapman Kolmogorov equations, definition of the generator, forward and backward equations
- Solution of forward equation by matrix exponential, expansion of <pi_n | in terms of exponentials of eigenvalues
- State and explain Master equation
- Sketch a sample paths of a CTMC and explain holding times, jump times, jump chain and their properties
- Stationary/reversible distributions, detailed balance, eigenvectors of generator matrix
- Result on existence of stat. distributions and uniqueness, irreducibility
- Poisson process, thinning and addition of PP (hand-out 2, page 2)
- birth-death chain, existence and formula for stationary distributions
- Definition of ergodicity and sufficient condition, state ergodic theorem
- Time-reversal of stationary Markov chains, reversibility, simple examples (RWs)
- Return time, transience, null/positive recurrence, explosion, simple examples

Interacting particle systems (IPS)

- Definition (state space and generator) of general IPS
- Define contact process, basic properties such as absorbing states, long-time behaviour, state mean-field rate equation and interpretation (Q2.2)
- Define voter model, basic properties such as absorbing states, stationary distributions
- Define exclusion process, conserved number of particles, irreducibility on sub-state space
- Explain Gillespie algorithm and random sequential update (hand-out 3)
- Explain how to measure the critical infection rate in the contact process (Q2.3)

Graph properties

- Definition, directed, undirected, adjacency matrix
- Definition of paths, distance, characteristic path length, diameter, connected components, compute for simple example
- In/out degree sequence, degree distribution, average, variance, compute for simple example
- Simple examples of graphs, complete, regular lattices, trees (characterizations)
- Degree correlations, edged biased sampling, definition of (dis)assortative, uncorrelated
- Definition of subgraph, cliques, spanning trees, simple examples
- Definition of clustering coefficients, compute for simple examples
- Definition of spectral density and graph laplacian
- Explain multigraphs, hypergraphs, bipartite graphs, multilayer networks

Random graph models

- Define E-R graphs, distribution of number of edges, expected degree distribution (binomial -> Poisson), explain local tree-like topology, compute clustering coefficient
- Define percolation on a graph, open subgraph, connected components
- State result on giant components for E-R random graphs, sketch relevant plots (Q2.5)
- State the Wigner semi-circle law, application to adjacency matrix of E-R graph, Perron-Frobenius Eigenvalue
- Explain preferential attachment mechanism, define Barabasi-Albert model, state basic properties of degree distribution, generalization of Q2.4, sketch relevant plots, non-linear generalizations
- Define small-world property and Watts-Strogatz random graphs, explain small-world regime in terms of average path length and clustering coefficient
- Define configuration model, simple examples including non-graphical sequences, sufficient condition for giant components of randomized graphs

Planar graphs and spatial point processes

- Define planar graph, faces, dual graph, draw simple examples including non-planar graphs, definition of triangulation and basic properties
- Define spatial point process and Poisson point process (PPP), explain how to sample from it
- Define Voronoi tessellation, draw for simple example, Delaunay triangulation

Examinable material.

includes the full lecture notes, all problem sheets and hand-outs, **EXCEPT:**

- Any facts on measurability/measure theory mentioned in the notes
- formulas on bottom of page 42 in the notes
- page 1 of hand-out 2 (generating functions)