Trade-Off between Efficiency and Fairness

A MathSys Project Proposed by Professor Bo Chen

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1 Introduction

We consider two key issues in a resource allocation problem: efficiency and fairness. It is clear that these two objectives are very often in conflict. As a first step to research on the trade-off between efficiency and fairness, we study the so-called *price of fairness* in a resource allocation problem that involves multiple self-interested parties or players and a central decision maker. We address the following question: How much is the relative system efficiency loss under a fair allocation assuming that a fully efficient allocation is one that minimizes the sum of player disutilities?

2 The Problem

We study the price of fairness (PoF) in scheduling $n = n_1 + n_2$ jobs on a single machine with n_1 and n_2 jobs owned by agents 1 and 2, respectively. The objective of each agent is to minimize the sum of completion times of his jobs, denoted by $f_1(\cdot)$ and $f_2(\cdot)$, respectively. The global objective is to minimize $f(\cdot) := f_1(\cdot) + f_2(\cdot)$. The PoF of a problem instance is the ratio of the (global) objective of a fair schedule and that of a (globally) optimal schedule, while the PoF of the problem is the supremum of the PoFs of all problem instances, as similarly defined in [2].

3 Minimax Fairness

Under minimax fairness, a fair schedule, denoted by S^{F}, should be one that minimizes the maximum of the two relative costs:

$$S^{\mathrm{F}} = \arg\min_{S} \max\left\{\frac{f_1(S)}{f_1^*}, \frac{f_2(S)}{f_2^*}\right\}, \tag{1}$$

where f_1^* and f_2^* are the minimum achievable objective values for agents 1 and 2, respectively.

It has been proved in [1] that the PoF for the problem is at least 2. They have also established that this bound is tight at least in some special case.

Lemma 1 *For any instance that satisfies*

$$(f^* - f_1^* - f_2^*)(\max\{f_1^*, f_2^*\} - \min\{f_1^*, f_2^*\}) \leq \min\{f_1^*, f_2^*\}(f_1^* + f_2^*), \quad (2)$$

the PoF is at most 2 *under the min-max fairness.*

Note that if $f_1^* = f_2^*$ or $f^* \leq f_1^* + f_2^* + \min\{f_1^*, f_2^*\}$, then condition (2) is satisfied.

4 The Project

Objective and methodology: The objective of this proposed project is to look beyond Lemma 1. Two approaches to the study are envisaged: empirical and analytical, and the project can take either or both of these approaches. An empirical study is to computationally evaluate an upper bound for the PoF on numerical instances of the scheduling problem that are generated according to some randomness scheme. An analytical study is to establish something similar to Lemma 1 but for a more general class of problem instances.

Deliverable: Establish either empirically or analytically an upper bound for the PoF for the problem or for a class of problem instances.

References

[1] A. Agnetis and B. Chen (2014). Bounds on the Price of Fairness for Single-Machine Scheduling. Working Paper.

[2] D. Bertsimas, V.F. Farias, and N. Trichakis, The Price of Fairness, *Operations Research* 59(1) (2011), 17–31.