Large scale dynamics in interacting particle systems

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Interacting particle systems are stochastic models of systems with a huge number of interacting components, such as states of matter or transport phenomena. These systems can exhibit interesting phase transitions as parameters, such as the total particle density, vary. Two such transitions, which are currently attracting significant research interest, are the condensation transitions in driven diffusive systems, and the glass transition which is observed in amorphous materials. The condensation transition is characterized by a finite fraction of all particles in the system concentrating in a small volume or a single lattice site. The glass transition is fundamentally dynamical in its characterization and is identified by an ergodicity breaking transition, huge relaxation times close to the critical point, dynamic heterogeneity (non-trivial spatio-temporal fluctuations of the local relaxation to equilibrium) and aging. In both of these cases we often know the stationary long time behavior of the system. However it is important, and often very challenging, to characterize the relaxation toward the stationary state. There are several project opportunities in this area, for example:

Project 1. A prototypical class of interacting particle systems which display many of the key dynamical features of real glassy materials are kinetically constrained models (KCM) [1].There are currently several conjectures on the relaxation and mixing properties of these glassy and condensing systems, which are not even supported by any significant numerical or simulation based evidence [2, 3]. In a first step existing novel techniques for simulating such process such as those in [4], could be applied to these models in order to support these conjectures. As a second step we would look at new numerical approaches for approximating the spectral gap (and associated eigenvectors) of these systems.
A student working in this area would also develop a good understanding of the current state of the art in mixing and relaxation of Markov interacting particle systems and could work toward theoretical support of these conjectures in parallel with simulation/numerical investigation.

Project 2. Equilibration dynamics of condensing systems have been fairly well understood recently for a particularly simple class of processes with zero-range interaction [5], and are currently under investigation for processes with more general dynamics [6], including for example inclusion processes [7]. Depending on the rates, these processes can also exhibit explosive condensation, a phenomenon similar to gelation, where the stationary state is reached in a time that vanishes with increasing system size.
The stationary current and its behaviour at and above the critical density is an important characteristic for the large scale dynamics of these models, and can be studied very well numerically with the help of recursion relations. In a project, different cases including divergence at the critical point, or convex/conave behaviour can be studied where recursion results are corroborated by simulation data and theory. This will provide an interesting introduction to simulation techniques and theoretical connections between microscopic particle systems and large scale emergent behaviour in such systems.

Note on availability and continuation: Paul may be away at conferences during part of the summer (certainly the end of July and start of August). Stefan is currently on leave and will only be available occasionally. If the project is successful, there may be the opportunity to continue to

PhD (subject to approval by MathSys), but Paul can not guarantee being in Warwick to supervise a PhD longterm.

References

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