Estimation of systemic risk in a model financial system

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Scientific background

Since the most recent financial crisis, there has been an increased appreciation by academics and policy-makers that effective regulation of the financial system requires an understanding of the complex dynamics underpinning its behaviour [1, 2]. A big issue is the management of systemic risk. The concept of systemic risk expresses the fact that the inter-dependence between financial institutions creates risks which are not easily visible at the level of individual institutions. Furthermore, these risks can be large. There doesn't seem to be an accepted quantitative definition of systemic risk: some authors characterise it as "risk of default of a large portion of the financial system". A notable attempt to quantify this is the notion of "DebtRank" metric proposed in [3]. The objective of this project is to define a model-independent quantitative proxy measure of systemic risk which can be estimated purely from empirical data and to test it on a toy model financial system where the amount of inter-dependence between institutions can be tuned.

Research challenge

At the level of individual institutions, there is a well developed empirical framework for quantification of risk. The most popular tools are Value-at-Risk (VaR models. See [4] for a review. Given a time-series of returns, $r(t)$, the risk of an individual institution's portofolio is estimated at a specified quantile q in the VaR methodology by estimating the size of the loss R which is exceeded a fraction $1-q$ of the time:

$$\mathbb{P}(r(t) > R) = 1 - q \tag{1}$$

where $\mathbb{P}(x)$ is the empirical return distribution. If there are 2 institutions, we propose the following definition of the systemic risk: the systemic risk is the risk that both institutions exceed their VaR threshold which is *not* accounted for by their individual risk levels. Denoting the returns on the portfolios of bank A and bank B by $r_A(t)$ and $r_B(t)$ and their corresponding VaR thresholds at some quantile q by R_A and R_B, this is quantified by

$$\rho = \mathbb{P}(r_A(t) > R_A, r_B(t) > R_B) - (1-q)^2, \tag{2}$$

where $\mathbb{P}(x, y)$ is the joint distribution of the returns of the two portfolios. There are (at least) two things wrong with this model:

1. Estimation of joint probability distributions suffers from the curse of dimensionality as the number of variables grows. It therefore becomes increasingly difficult to sample the space as the number of institutions grows.
2. This is compounded by the fact that we are typically interested in high quantiles of the returns distribution (systemic crises are rare events). As the targeted quantile increases the tails of empirical probability distributions may become increasingly a-typical.

This project has three strands:

- **Generation of model data:** Generate a small toy network model of a financial system with small numbers of institutions (nodes) and define model dynamics on that network which contain a parameter which tunes the degree of interdependence between the portfolios of the nodes.
- **Testing of systemic risk measures:** Test the metric 2 above on synthetic data from the model system to establish how far it can be pushed and remain statstically significant.
- **Improve the systemic risk measure**: can the shortcomings of Eq. (2) be addressed using more sophisticated thinking?

Additional considerations

This project is reserved for Max Smilovitsky. If the project goes well it is hoped that it would provide a springboard for closer collaboration with Sciteb possibly in the form of a PhD project in the area of financial regulation.

References

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