Social Pressure in Coevolving Voter Models

MSc Project Proposal by G. Mosquera & S. Johnson

Outline

An emerging field in Complexity Science is that of Coevolving or Adaptive Networks (CN), where the dynamics of the topology of a system and its underlying interacting process are coupled. CN have been applied to Voter Models (VM), leading to the so-called Coevolving Voter Model (CVM) [1, 2, 3]. In CVM the existing connections of the graph are rewired at certain time scale and certain probability, usually following a rewire-to-same scheme where active links (connecting nodes with opposite opinion) tend to be substituted by frozen links (connecting nodes with same opinion). The CVM has absorbing states where different opinions coexist due to a fission of the graph.

On the context of Social Networks, it is reasonable to think that influence/reputation (i.e. connectivity) should be granted to those nodes having better performance in some measurable sense. For example, when considering the VM as a system of traders buying or selling in a speculative market, we should be able to monitor the profits achieved by every node given its current position. This would require us to define some Price parameter, which should be derived from the proportion of buyers/sellers in the system. Assuming this could be done, we could then couple the earnings of each agent to its rewiring probability (the probability of establishing links connecting him with other nodes). In particular, it would be reasonable for the probability of connecting two nodes to be proportional to the difference in the earnings of each one. The investigation of this modified CVM and the properties of the resulting network (mainly degree distribution, assortativity, clustering, dimensionality and scaling) is one of the fundamental objectives of the project.

The above description of the "earnings" process suggests considering the VM as some sort of game where agents obtain a social utility depending on their actions and those of the rest of the system. A natural scenario is the already introduced stock market environment, where social utility would be the net profit for each agent. In this "gaming" context, it is natural to think of an extended version of the VM where agents have an alternative strategy to herding. The weight of each strategy would be determined by the "social pressure" on each agent, a parameter measuring the amount of incoming influence. An individual subject to large social pressure will tend to rely more heavily on herd behaviour rather than any other rational strategy. Labelling the binary state of opinion by $n_i \in \{0, 1\}$, the probability of adopting state 1 for an activated node i would be:

$$P_1^i(t) = S_i(t)\langle n_j(t)\rangle_{j\in\nu_i} + (1 - S_i(t))\mathcal{U}_i(t) \tag{1}$$

where $S_i(t) \in [0, 1]$ is the social pressure parameter for agent i at time t, $\langle n_j(t)\rangle_{j\in\nu_i} \in [0, 1]$ is the weighted average opinion of the neighbourhood ν_i of node i, and $\mathcal{U}_i(t) \in [0, 1]$ is a function representing an alternative strategy to herding.

1

It is reasonable to think that $S_i(t)$ will be a function of the in-degree (number of incoming influencers) and out-degree (outgoing influence) of node i as well as the total out-degree of its neighbourhood, i.e. $S_i(t) = f(k_i^{in}(t), k_i^{out}(t), \sum_{j \in \nu_i} k_j^{out}(t))$. In particular, we propose:

$$S_i(t) = 1 - e^{-x}, \quad \text{where} \quad x = k_i^{in}(t) \frac{\sum_{j \in \nu_i} k_j^{out}(t)}{C + k_i^{out}(t)} \tag{2}$$

where C is a real parameter of the model, which modulates the intensity of the Social Pressure phenomena.

A first approximation to "rationality" term $\mathcal{U}_i(t)$ could be the zero-intelligence model $\mathcal{U}_i(t) \sim U(0,1)$, although more elaborated forms of $\mathcal{U}_i(t)$ should be investigated. In particular, it would be desirable to find a strategy that maximizes expected profits given the amount of available information to each agent.

An analytical and numerical study of the properties of this modified version of the VM with Social Pressure is the other main goal of the project.

Background and Methodology

A reasonable knowledge of Interacting Particle Systems and Network Theory is required for the development of the theoretical frame of this project. In addition, extensive use of computational methods (mainly MC simulation) is going to be essential.

Prospective Deliverables

The project have two building blocks, namely the Coevolving Network and the modified version of the VM. They could be standalone investigations, although the most ambitious goal is to merge both concepts (i.e. study the Social Pressure VM in the "earnings" based Coevolving Network). Both problems can be approached with increasing levels of sophistication, thus avoiding all-or-nothing outcomes.

Possible Data Sources

This is a theoretical project, where research should be based on analytical derivations as well as computer simulations. Despite this, in the most optimistic scenario the Coevolving Network could be compared to real data from social networks (e.g. Twitter).

Further Extensions

This project will hopefully lead to new results on the field of Opinion Dynamics. More complex approaches to the problems presented here can be easily developed. For example,

the process of Price Formation is susceptible to further investigations, with the objective of finding a rigorous link between Opinion Dynamics and Price Dynamics. Therefore, this MSc project can be extended into a PhD topic.

References

[1] R. Durrett et al., "Graph Fission in an Evolving Voter Model". *PNAS.* **109** (2012)

[2] M. Diakonova, V.M. Eguíluz, M. San Miguel, "Noise in Coevolving Networks", *Physical Review E* **92**, 032803 (2015)

[3] F. Vazquez, V.M. Eguíluz, M. San Miguel, "Generic Absorbing Transition in Coevolution Dynamics", *Physical Review Lett.* **100**, 108702 (2008)