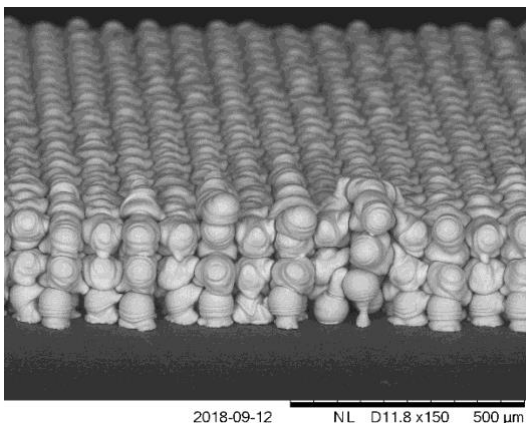


# Mathematical modelling of 3D printing processes

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Additive manufacturing (AM), popularly known as 3D printing, is an approach that allows creation of complex objects by building them gradually, usually voxel by voxel. While most consumer 3D printers work by extruding (squeezing out) a polymer, much like toothpaste, that then solidifies, a much wider variety of AM technologies are used in industry or being investigated. Some of them potentially allow multimaterial 3D printing, with mind-boggling applications, such as printing all parts of an electronic device (a plastic body, electronic components like transistors, and metal wires connecting them) in a single process.

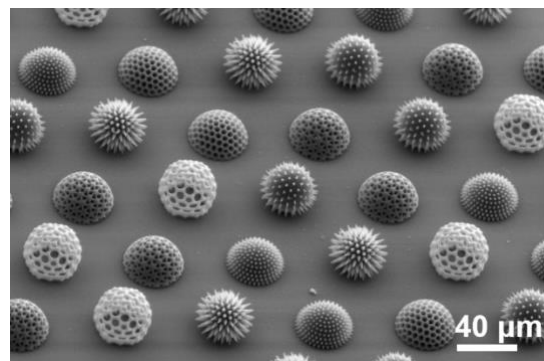
For this project, you will have a choice of modelling one of two AM processes. **Metaljet** is a variant of the more common inkjet technology in which instead of a dye or a polymer, molten metal drops are deposited one by one and solidify, forming an object. One of the challenges, particularly for multimaterial metaljetting, is ensuring that the newly deposited drop “sticks” to those deposited previously or to the substrate. In a recent paper [1] it was shown experimentally that depending on the substrate material, a molten tin drop can stick to the substrate or “self-peel” off of it due to mechanical stresses developing as the solidified drop cools down. In the project, you will simulate this process using linear thermoelasticity and fracture mechanics. If time permits (or if you decide to do a PhD), you can go much deeper, also modelling the fluid dynamics of how the drop spreads while solidifying.



Solidified silver drops deposited by the Metaljet process. Photo by Nesma Aboulkhair (Nottingham)

In **two-photon polymerisation** (2PP) [2], one starts with a liquid sample that is solidified by a focused laser beam. As the name suggests, solidification (or “curing”)

is due to a polymerisation reaction that is only initiated when two laser light photons are absorbed by the same photoinitiator molecule at the same time, which requires high light intensities only found at the very centre of the focus. This gives extremely high, submicron resolution and allows creation of small but very complex objects with tiny features. However, laser intensity needs to lie within a sometimes narrow “processing window”: if the intensity is too low, not enough curing takes place to make a solid, and if it is too high, overheating boils the liquid and damages the object. The processing window depends not only on the material, but also the exact details of the laser scanning strategy, such as how fast the beam moves, how many times a given volume is exposed, and the distance between adjacent lines that have been exposed. To help understand this and predict the properties of the resulting material, you will develop a mathematical model of 2PP including coupled processes of light absorption, chemical reactions, diffusion, and heat transport; it may even be possible to use it to guide the manufacturing process in real time.



Tiny objects produced by 2PP. Photo by Qin Hu (Nottingham)

Working on this project, you will be a part of a large collaboration including both experimentalists and modellers, working on different approaches to multimaterial AM [3]. There are already plenty of data available, particularly for 2PP, to help guide your modelling effort, and you will be able to request more data if necessary. It will be possible for you to participate in regular research meetings in which new ideas for collaboration are likely to arise.

[1] J. de Ruiter *et al.*, *Nature Phys.* 14 (2018) 35

[2] C. N. LaFratta and T. Baldacchini, *Micromachines* 8 (2017) 101

[3] <http://www.nottingham.ac.uk/NextGenAM>