Parameter estimation and model selection for crowd dynamics

Susana Gomes & Marie-Therese Wolfram (Mathematics Department)

Motivation

Understanding the behaviour of crowds is crucial for safety and transport management: on the one hand, it allows to predictions of people's behaviour (e.g., what their reaction would be in an evacuation), and on the other hand it facilitates better design of rooms, buildings or even streets to optimise movement. There are many types of models for pedestrian dynamics, ranging from microscopic models (agent-based models that describe the trajectory of an individual pedestrians) to kinetic models to macroscopic ones (a PDE model describing the density of the crowd). Most models in the literature rely on the fundamental diagram: a tool that engineers use to characterise the speed of pedestrians to the average density they experience. Although there is a general agreement on the shape of this function, its parametrisation depends strongly on the measurement and averaging techniques used as well as the experimental setup considered. The goal of this project is to improve our understanding of this function, and to quantify how well different models (involving different functional forms of the fundamental diagram) describe real-life behaviour.

The project

We will consider the unidirectional flow of a crowd in a corridor. In this case the movement of pedestrians can be characterised by a simple ODE or SDE whose speed (drift term) is given by the fundamental diagram:

$$d\boldsymbol{X}(t) = F(\rho(X(t), t)\; dt + \sigma dW(t),$$

where $\boldsymbol{X}(t) = (x(t), y(t))$ is the position of a pedestrian, ρ is the density, $F(\rho)$ is the fundamental diagram, and $W(t)$ is a two-dimensional white noise with strength σ. This model is connected to a PDE model for the density,

$$\partial_t \rho = \nabla \cdot (\sigma \nabla \rho - F(\rho)\rho),$$

where both equations are coupled with appropriate boundary conditions. This project will build upon the supervisors' previous work on parameter estimation for this problem, where they used a simple form of the fundamental diagram [1] and expand this methodology to more realistic forms of the fundamental diagram (see, e.g., [2]) and incorporate model selection techniques such as the Bayes factor [3]. This can be done using artificial data (generated from the various models) as well as data from experiments. Depending on the student's background and preferences, we will explore deterministic and Bayesian techniques, and the student will also have the opportunity to explore likelihood-free inference techniques such as Approximate Bayesian Computation (ABC) [4].

Available data and resources

The student will have access, if needed, to code from the supervisors (written in Python and/or Matlab), to solve both the ODE/SDE problem, as well as the PDE in two dimensions. Furthermore, we will also use experimental data from collaborators from the Institute for Advanced Simulation (IAS) in Julich, Germany.

References

[1] S.N. Gomes, A.M. Stuart, and M.T. Wolfram, *Parameter estimation for macroscopic pedestrian dynamics models from microscopic data,* SIAM Journal on Applied Mathematics, 79(4):1475-1500, 2019.
[2] M. Twarogowska, P. Goatin, and R. Duvigneau, *Comparative study of macroscopic pedestrian models*, Transportation Research Procedia 2: 477-485, 2014.
[3] N. Bode and E. Ronchi, *Statistical model fitting and model selection in pedestrian dynamics research*, Collective Dynamics 4, A20:132, 2019.
[4] J. Lintusaari, M. U. Gutmann, R. Dutta, S. Kaski, and J. Corander. *Fundamentals and recent developments in Approximate Bayesian Computation*. Systematic biology, 66(1):e66–e82, 2017.