1+1=2? Well, think again...

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Empirical Motivation

- Boyson et al. (2010) analysed data on the returns of eight different Hedge Fund styles from January 1990 to October 2008.
- They concluded that the worst hedge fund returns, defined as returns that fall in the bottom 10% of a hedge fund style's monthly returns, show higher correlation than expected from economic fundamentals (contagion).

Theoretical Insight

- Contagion is linked with liquidity shocks, in support for the mechanism proposed by Brunnermeier and Pedersen (2009).
- Brunnermeier and Pedersen (2009) links an asset's market liquidity and traders' funding liquidity.
- Traders provide market liquidity, and their ability to do so depends on their availability of funding.
- Conversely, traders' funding, depends on the assets' market liquidity.
- Thus, there is a reinforcing mechanism at play leading to liquidity spirals.

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Relevant Literature

- The mechanism proposed by Brunnermeier and Pedersen (2009) is related to the so-called "Leverage Cycle"(Geanakoplos, 1997, 2010a,b; Thurner et al., 2012; Poledna et al., 2014).

The leverage cycle in a nutshell:

The Leverage Cycle

The pro-cyclical expansion and contraction of credit supply.

- Leverage becomes too high in boom times, and too low in bad times.
- As a result, in boom times asset prices are too high, and in crisis times they are too low.

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1. The link between heterogeneity and the clustering of defaults.
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The Economy

- Traders have a choice between owning a risky and risk-free asset.
- Two kinds of traders:
  1. Noise traders.
  2. Hedge funds (HF). (Receive a private noisy signal. Signal precision varies among HFs).
- Credit: The HFs can increase the size of their long position by borrowing from a bank using the asset as collateral.

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Key Results

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    1. Microscopic level: Exponentially distributed $\Rightarrow$ Poisson process.
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Fat-tail, so what?

Consequences of the fat-tail

- The emergence of a fat-tailed distribution of WTBD on the aggregate level leads to clustering of defaults.
- The bursty character of the occurance of defaults allows a *deterministic* description of the time-sequence of defaults.

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Noise Traders

- The demand d^{nt} of the representative noise-trader for the risky asset, in terms of cash value, is assumed to follow an AR(1) mean-reverting process (Xiong, 2001; Thurner et al., 2012; Poledna et al., 2014).
- Thus, the demand (in cash value) $d_t^{nt} = D^{nt} p_t$ of the NTs follows

$$\log d_t^{nt} = \rho \log d_{t-1}^{nt} + \sigma^{nt} \chi_t + (1 - \rho) \log(VN). \quad (1)$$

where $\chi_t = N(0,1)$ and $\rho \in (-1, 1)$.

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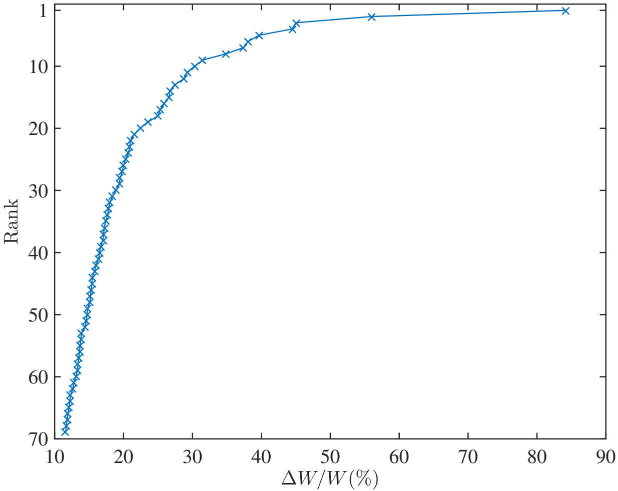
Hedge Funds I



Hedge Funds II

- HFs are represented by risk averse agents with CRRA.
- Utility: $U = 1 - e^{-\alpha r_t^j}$, where r_t^j denotes the rate of return of the jth HF, i.e. $r_t^j = (W_t^j - W_{t-1}^j)/W_{t-1}^j$.
- Each HF receives a private noisy signal $\tilde{V} = V + \epsilon_j$.
  - V the fundamental value of the risky asset.
  - $\epsilon_j \sim \mathrm{N}(0, \sigma_j^\epsilon)$.

Their wealth at each period is $W_t^j = D_t^j p_t + C_t^j$.

- D_t^j, demand for the risky asset.
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Funds

The maximization yields

$$D_t^j = \frac{m}{\alpha\sigma_j^2} W_t^j, \quad m = V - p_t. \tag{2}$$

- Demand is capped by $\lambda^j = D_t^j p_t / W_t^j \leq \lambda_{\max}$,

$\lambda_{\max}$ the maximum allowed leverage set externally.

Price

- The wealth of a HF evolves according to

$$W_{t+1}^{j} = W_t^j + (p_{t+1} - p_t)D_t^j - F_t^j \tag{3}$$

- F_t^j, managerial fees following the 1/10 rule:

$$F_t^j = \gamma\left(W_{t-1} + 10\max\left\{W_{t-1}^j - W_{t-2}^j, 0\right\}\right) \tag{4}$$

- The price of the risky asset is determined by the market clearance condition

$$D_t^{\text{nt}}(p_t) + \sum_{j=1}^{n} D_t^j(p_t) = N. \tag{5}$$

What is Clustering?

If defaults are clustered, then $C(t')$ decays such that the sum of the autocorrelation function over the lag variable diverges (Baillie, 1996; Samorodnitsky, 2007). Thus,

Definition

Let $C(t')$ denote the autocorrelation of the time series of defaults, with t' being the lag variable. Defaults are clustered iff

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Theorem

Consider an exponential density function $P(\tau;\mu)$, parametrized by $\mu \in \mathbb{R}_+$. If μ is itself a random variable with a density function $\rho(\mu)$, and $\rho(\mu)$ in a neighbourhood of 0 can be expanded in a power series of the form $\rho(\mu) = \mu^\nu \sum_{k=0}^{n} c_k \mu^k + R_{n+1}(\mu)$, where $\nu > -1$, then the leading order behaviour for $\tau \to \infty$ of the aggregate probability function is $\tilde{P}(\tau) \propto \tau^{-(2+\nu+k)}$, where k is the order of the first non-zero term of the power series expansion of $\rho(\mu)$ for $\mu \to 0_+$ (exhibits a power-law tail).

Proof.

The aggregate density can be viewed as the Laplace transform $\mathcal{L}[.]$ of the function $\phi(\mu) \equiv \mu W(\mu)$, with respect to μ. Hence,

$$\tilde{P}(\mu) \equiv \mathcal{L}[\phi(\mu)](\tau) = \int_0^\infty \phi(\mu) \exp(-\mu\tau) d\mu. \quad (7)$$

Watson's Lemma (Debnath and Bhatta, 2007):

$$\mathcal{L}_\mu [f(\mu)](\tau) \sim \sum_{k=0}^{n} b_k \frac{\Gamma(a+k+1)}{\tau^{a+k+1}} + O\left(\frac{1}{\tau^{a+n+2}}\right). \quad (8)$$

Therefore,

$$\tilde{P}(\tau) \propto \frac{1}{\tau^{k+\nu+2}} + O\left(\frac{1}{\tau^{k+\nu+3}}\right). \quad (9)$$

□

Autocorrelation

Theorem

Let $T_n \in \mathbb{R}_+$, $n \geq 0$, *be a sequence of i.d.d. random variables. Assume that the probability density function* $\tilde{P}(T_n = \tau) \propto \tau^{-\alpha}$, *for* $\tau \to \infty$.

Consider now the renewal process $S_n = \sum_{i=0}^{n} T_i$. *Let* $Y(t) = 1_{[0,t]}(S_n)$, *where* $1_A \; : \; \mathbb{R} \to \{0,1\}$ *denotes the indicator function, satisfying*

$$1_A = \begin{cases} 1 & : \quad x \in A \\ 0 & : \quad x \notin A \end{cases}$$

If $2 < \alpha \leq 3$, *then the autocorrelation function of* $Y(t)$, *for* $t \to \infty$ *decays as*

$$C(t') \propto {t'}^{2-\alpha} \tag{10}$$

Proof.

A renewal process is ergodic:

$$C(t') \propto \lim_{K\to\infty} \frac{1}{K} \sum_{t=0}^{K} Y_t Y_{t+t'}. \tag{11}$$

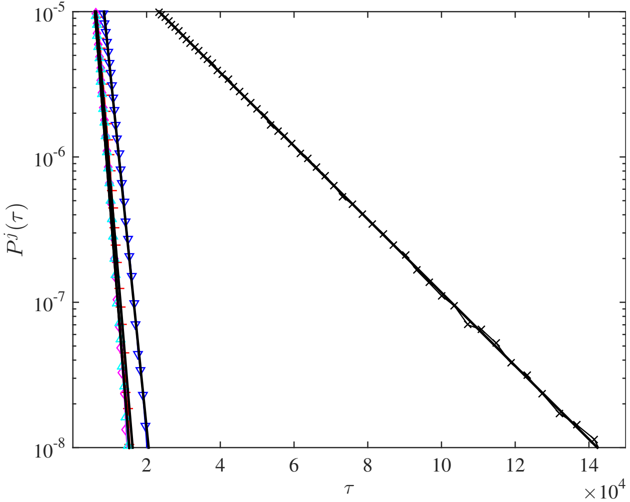
The correlation function can then be expressed in terms of the aggregate density (Procaccia and Schuster, 1983; Schuster and Just, 2006):

$$C(t') = \sum_{\tau=0}^{t'} C(t'-\tau)\tilde{P}(\tau) + \delta_{\tau,0}. \tag{12}$$

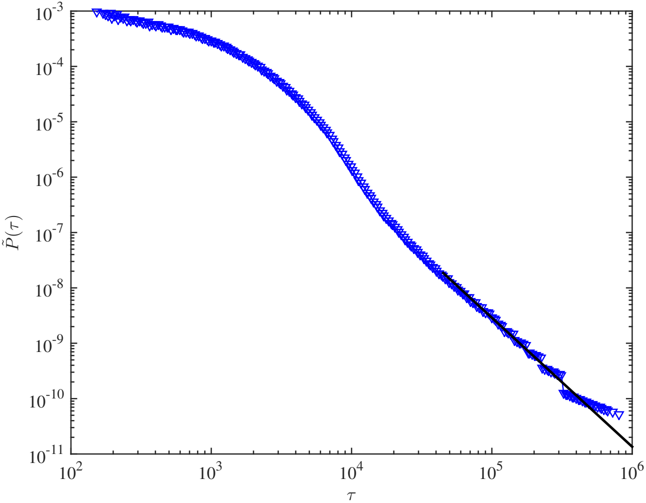
$$\mathcal{F}\{C(t')\} \overset{f\ll 1}{\propto} \begin{cases} f^{a-3}, & 2<a<3 \\ |\log(f)|, & a=3 \\ \text{const.}, & a>3 \end{cases}. \tag{13}$$

□

Failure Function — Microscopic Level



After Aggregation



Clustering of Defaults

Asymmetric and Information Leads to Clustering of Defaults

An important effect of the emergent heavy-tail statistics stemming from the heterogeneity of the market, is the absence of a characteristic time-scale for the occurrence of defaults (scale-free asymptotic behaviour).

- Fitting the aggregate distribution we obtain $\tilde{P}(\tau) \sim \tau^{-(7/3)}$.
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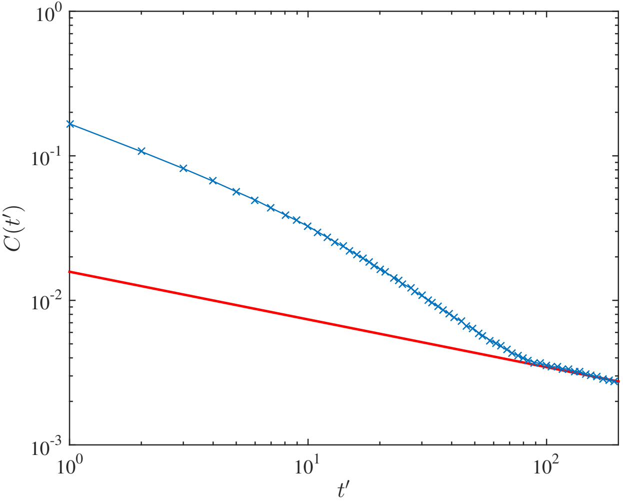
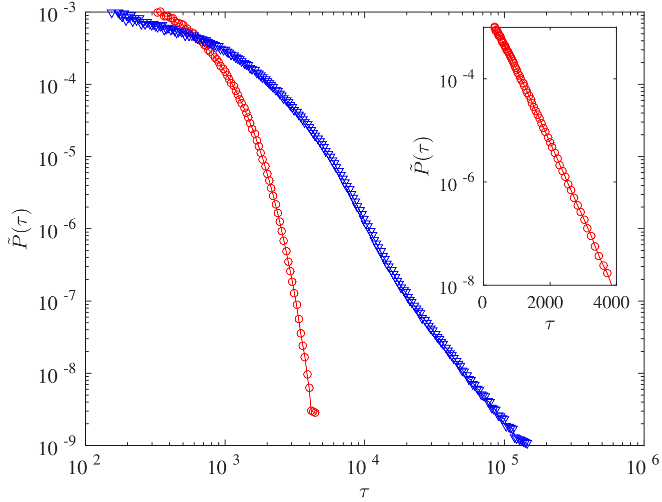
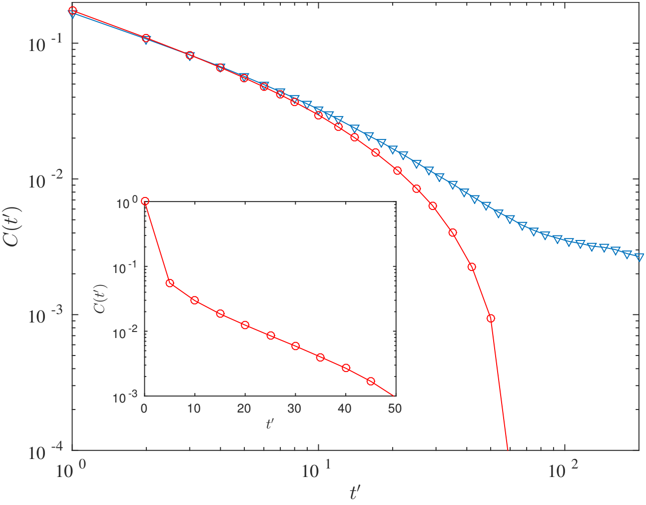
Autocorrelation Function



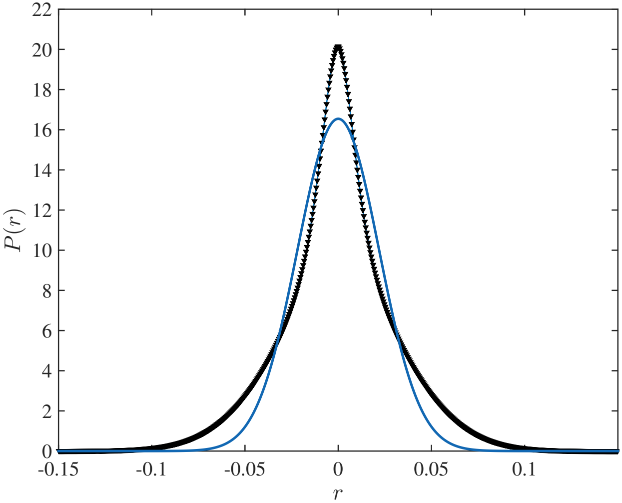
Better Information for All



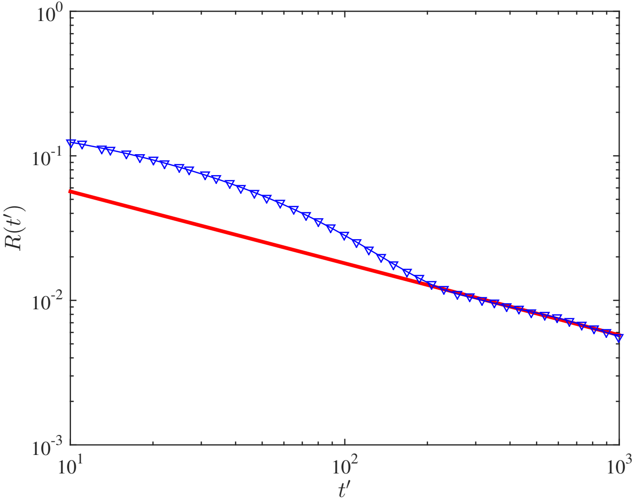
Numerical results



Non-Normal Returns



Clustered Volatility



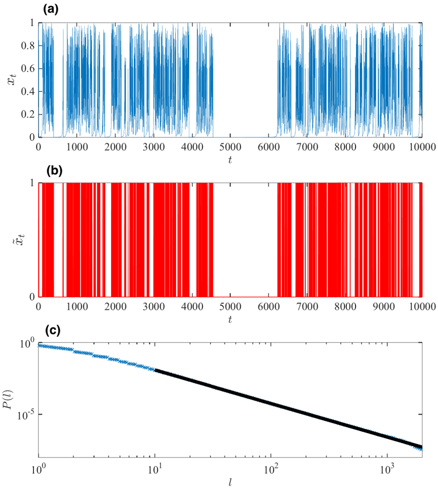
Deterministic Description

- All statistical properties of default events can be replicated by a very simple deterministic map.

$$x_{t+1} = x_t + u x_t^z \mod 1, \; z > 1. \quad (15)$$

- Characteristic behaviour: The evolution of x_t is regular close to the vicinity of 0 (marginally unstable fixed point) and chaotic away from it $\Rightarrow$ Random alternation between almost regular and chaotic dynamics.
  - Regular motion $\rightarrow$ Laminar phase.
  - Chaotic motion $\rightarrow$ Turbulent phase.

Deterministic Description II



Deterministic Description III

- The distribution of waiting times between transition from the laminar to the turbulent phase follows a power-law (Schuster and Just, 2006).

$$\rho(\tau) \propto \tau^{-\frac{z}{z-1}}, \tag{16}$$

- Also, the autocorrelation function of x_t decays algebraically

$$C(t') \propto t'^{\frac{z-2}{z-1}},\ 3/2 \leq z < 2. \tag{17}$$

Setting $z = \frac{7}{4}$, and mapping the:
  - HFs Active $\rightarrow$ Laminar phase.
  - Default events $\rightarrow$ Turbulent phase.

$$\rho(t) \sim \tau^{-7/3},\ C(t') = t'^{-1/3} \tag{18}$$

- We assume that the heterogeneity of the agents stems from the HFs' different quality of the mispricing signals they receive.
- We show that the failure function of the HFs is qualitatively different when observed on the micro and the aggregate level.
- We also show that the scale-free property of the emergent statistics on the aggregate level is directly connected with the clustering of defaults.

Which is the Real Cause?

. . . A crucial part of my story is heterogeneity between investors. . . But an important difference is that I do not invoke any asymmetric information. . . Of course, the asymmetric information revolution in economics was a tremendous advance, and asymmetric information plays a critical role in many lender-borrower relationships; sometimes, however, the profession becomes obsessed with it. . . (Geanakoplos, 2010a)

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