Warwick Agents and Games in Economic Research

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February 11, 2016

Outline

WAGER

- Shared interest in economics topics
- Particular intersection of interests: many, heterogenous, interacting agents
- More specifically interests revolve around:
    - (Evolutionary) Game Theory
    - Networks
    - Opinion Dynamics
    - Interacting Particle Systems
    - Agent Based modeling

- Meet and share or discuss our research
- Format: fortnightly meeting?
- Not too formal (slides are not generally expected)
- Relevant references or other on-topics talks are welcome!

A word of warning...

The case for microfoundations

- Macroeconomics until the 70ies...
- ...fitting large polinomyials to time series (ehm, VERY ROUGHLY)
- But, people react to policy stance over time and adjust
- Aggregate relations might change
- Lucas (1976) Microfoundations are required
- Study “deep” parameters governing decision making (elasticities, risk aversion)
- Draw policy conclusions

The case for rationality

$$\text{How to describe agents behavior?} \tag{1}$$

Rationality, in some form, provides a direction to answer (2).

The word has many meanings in economics:

- Rational preferences: $\succeq: X \times X \rightarrow X$ is a complete preorder
- Agents act according to their preferences to maximize utility
- Full rationality: unlimited "computing power"
- Rational Expectations: agents choice coincide with the equilibrium prediction of the model

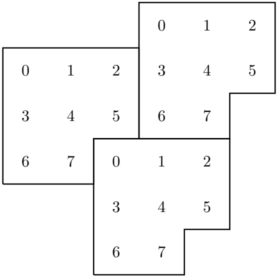
The case for microscopic interactions

$$\text{When does the aggregate differ from the whole?} \tag{2}$$

- Only for quasi-linear preferences (Gorman, 1961)
- Sonnenschein-Mantel-Debreu th.
- When do economic effects (incentive) trump other effects?
- Some of these questions may be cast in economic terms (spillover, externalities, role of informations)
- Often provide good "why"s, they might not be "actionable"

Potts-like models of fashions

- Agents $i \in \mathcal{I} = \{1, \ldots, N\}$
- Arrayed on lattice Λ with helical boundary conditions
- Neighbors of i, $B_i = \{j \in \mathcal{I} : (i \pm 1) \bmod N, (i \pm L) \bmod N\}$



Each chooses a style $x_i \in \{1, \ldots, q\}$ according to the utility function

$$V_i(x) = u(x_i) + S(x_i, x_{-i}) + \epsilon(x_i) \tag{3}$$

Agents choose according to the utility function:

$$V_i(x) = \underbrace{u(x_i)}_{\text{Private Utility}} + \underbrace{S(x_i, x_{-i})}_{\text{Social Utility}} + \underbrace{\epsilon(x_i)}_{\text{Noise}} \tag{4}$$

- $\epsilon(x) \sim GEV(0, \beta^{-1})$
- $f(x) = e^{e^{-\beta x}}$

Agent picks x over y if:

$$\mathbb{P}(V_i(x) > V_i(y)) = \mathbb{P}(\epsilon(x) - \epsilon(y) > K) = \frac{e^{\beta \bar{V}_i(x)}}{e^{\beta \bar{V}_i(x)} + e^{\beta \bar{V}_i(y)}} \tag{5}$$

Possible forms of $S(x_i, x_{-i})$:

- $J \sum_{j \in B_i} (x_i - x_j)$ Linear interactions
- $J \sum_{j \in B_i} \delta_{x_i, x_j}$ Potts
- $J \sum_{j \in B_i} x_i \tanh(x_i - x_j)$ Tanh

Markov process

- Pick agent with probability $\frac{1}{N}$
- Pick style with probability $\frac{1}{q}$
- Agent switch from y_i to x_i with prob. $\mathbb{P}(V_i(x) > V_i(y))$

This defines a Markov process $\{x_t\}_{t\geq 0} \in \prod_t \Omega_t = \{1, \ldots, q\}^\Lambda$

$$x^{jl} := \begin{cases} x_i^{jl} = x_i & , \forall i \neq j; \\ x_j^{jl} = l & othw. \end{cases}$$

Transition rates matrix ϕ:

- $\phi(x, x^{jl}) = \frac{\exp\{\beta \bar{V}_i(x^{jl})\}}{Nq}$
- $\phi(x, x) = -\sum_{j,l} \phi(x, x^{jl}) =: -\phi_x$
- $W_x := \inf\{t \geq 0 : x_t \neq x\}$
- $W_x \sim \exp(\phi_x)$

Stationary distribution

$$\mu(x) = \frac{\exp\{\beta \sum_i \bar{V}_i(x)\}}{\sum_{z \in \Omega} \exp\{\beta \sum_i \bar{V}_i(z)\}} \tag{6}$$

Proof by checking detailed balance, since reversibility implies stationarity:

$$\mu(x)\phi(x,y) = \mu(y)\phi(y,x) \tag{7}$$

Irreducibility implies that this is the unique distribution.

- Behavior of player is fully rational under heavy discounting
- Many boundedly rational alternative interpretations
- Distribution places most weight on configuration with highest “Bentham welfare”
-

Possible form of $S(\cdot)$

$$V_i(x) = \underbrace{u(x_i)}_{\text{Private Utility}} + \underbrace{S(x_i, x_{-i})}_{\text{Social Utility}} + \underbrace{\epsilon(x_i)}_{\text{Noise}} \tag{8}$$

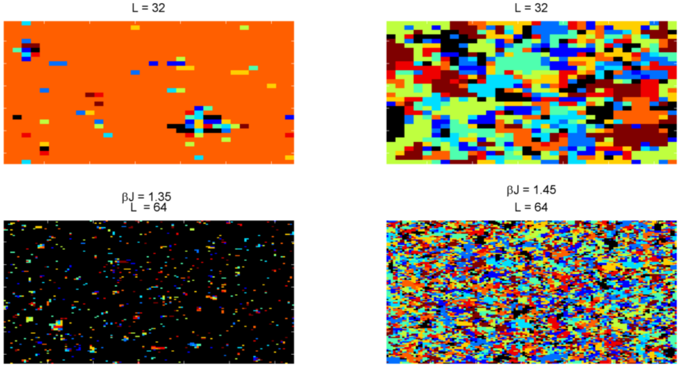
Pick extreme case: $u(x_i) = 0$

$$S(\cdot) = J \sum_{j \in B_i} (x_i - x_j) \tag{9}$$

- Places uniform weight on all configuration
- Baseline
- Interactions need to be non-linear

$$S(\cdot) = J \sum_{j \in B_i} \delta_{x_i, x_j} \tag{10}$$

- Favors alignment
- "Well-known"
- Order parameter $\langle q\delta_{x,y} - 1 \rangle$



Some questions:

- How does deviance emerge and how do deviants become the norm?
- What do the equilibria look like?
- Is opacity relevant?
- What can this model be applied to?
- Dynamics and time scales

Possible form of $S(\cdot)$

$$S(\cdot) = J \sum_{j \in B_i} x_i \tanh(x_i - x_j) \tag{11}$$

- Non-odd function
- People like to imitate “better” styles (higher x_i)
- Relative to where they are

Some references

- Blume, L. E. The Statistical Mechanics of Strategic Interaction.
  Games and Economic Behavior, 5(3):387–424, 1993

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  Econometrica, 61(1):57–84, 1993

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  MIT Press, 1992

- Kolokoltsov, V. The evolutionary game of pressure (or interference), resistance and collaboration.
  dec 2014

Why do chain beget chains? A Markov version

We turn Page and Tassier (2007) into a Markov model.

- Set of markets $\mathbb{K} = \{1, \dots, k, \dots, K\}$, locations with periodic boundary.
- Set of hedonic characteristics $\mathbb{L} = \{1, \dots, l, \dots, L\}$, tastes
- Firms have $\mathbb{Q} = \{0, 1, \dots, q, \dots, Q\}$

Transition rates:

$$\phi(x \to y = x^{lk}) =$$

$$(1 - \delta_{x_{lk}}) D_{lk}(y) \left[(KLQ)^{-1} + \delta_{x_{lk+1}, x_{lk}} + \delta_{x_{lk-1}, x_{lk}} \right] + \\ + \frac{\delta_{x_{lk}}}{D_{lk}(y)}$$

Demand is given by:

$$D_{lk}(y) = \frac{y_{lk} + 1}{1 + y_{l+1k} + y_{l-1k}} \tag{12}$$

Pricing on the line

Work in progress, together with Andrea Pizzoferrato

- Lattice $\Lambda = \{1, \ldots, N\}$
- Occupation number η_i
- Fixed(?) number of consumers N
- Reserve price $\bar{p}$

$$\phi(\eta, \xi) = \frac{1}{N}\Bigg[\delta_{\xi_i, \eta_i + 1}\left(\frac{N - \eta_i}{\bar{p} - p_i}\right)\sum_j \left(\frac{\bar{p} - p_j}{N - \eta_j}\right) + \delta_{\eta_i, \eta_i - 1}\left(\frac{\bar{p} - p_i}{N - \eta_i}\right)\sum_j \left(\frac{N - \eta_j}{\bar{p} - p_j}\right)\Bigg] \tag{13}$$

Each location is a shop, shop act on a different time scale (w.r.t. customers) and set p_i to maximize the flow of customer.

Next meeting: 25 February 2016, 5pm, D1.07. Speaker TBA

Thank you for your attention!

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