Analysing early warning signals of disease elimination by approximating the potential surface

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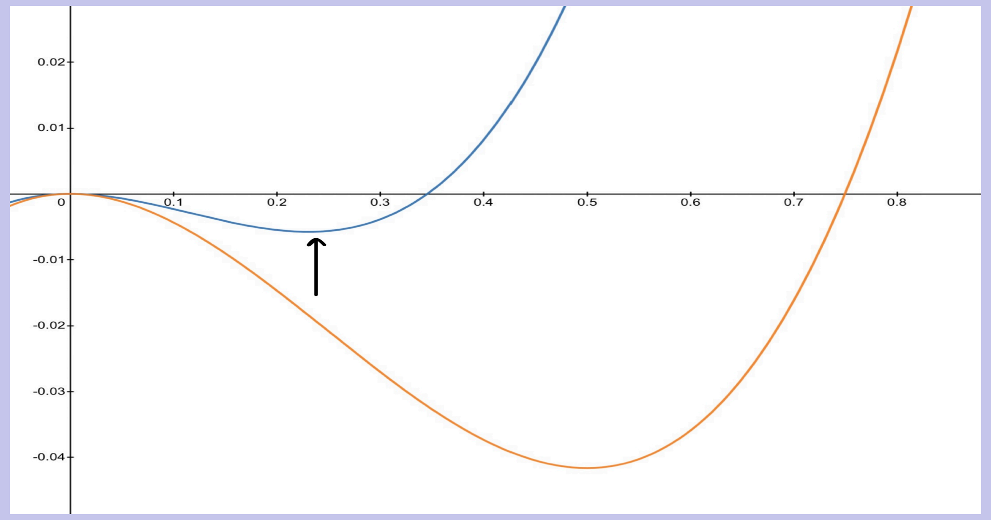
Theory of Early Warning Signals

Early warning signals are time-series statistics designed to detect an impending critical transition. There are two theories describing how early warning signals may behave as a critical transition is approached.

Critical Slowing Down [1]

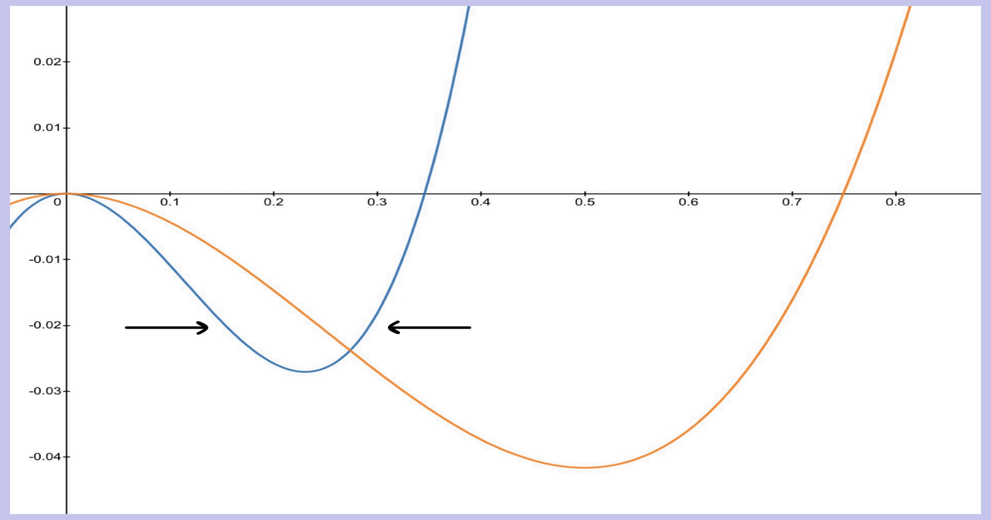
- The system becomes **slower** to return to equilibrium after stochastic disturbances.
- The potential surface becomes **flatter and shallower**.



Example Early Warning Signals:
- Autocorrelation **increases**
- Variance of fluctuations around the equilibrium **increases**.

Critical Speeding Up [2]

- The system becomes **faster** to return to equilibrium after stochastic disturbances.
- The potential surface becomes **steeper**, but not deeper.



Example Early Warning Signals:
- Autocorrelation **decreases**
- Variance of fluctuations around the equilibrium **decreases**.

Understanding which of these behaviours will occur is crucial to interpreting early warning signals, but may not be immediately clear given a particular model or type of data.
Studying the **change in the steepness of the potential surface** offers a method for identifying critical slowing down/speeding up.

1. Scheffer, Marten, et al. "Early-warning signals for critical transitions." *Nature* 461.7260 (2009): 53-59.
2. ,Titus, Mathew, and James Watson. "Critical speeding up as an early warning signal of stochastic regime shifts." *Theoretical Ecology* 13.4 (2020): 449-457.
3. Yates, Christian A., et al. "Inherent noise can facilitate coherence in collective swarm motion." *Proceedings of the National Academy of Sciences* 106.14 (2009): 5464-5469.

Application to the SIS Model for an epidemic

We derive analytic equations and use an equation-free approximation method of to study the potential surfaces of three types of data form the SIS model.

Analytic Method

$$S + I \xrightarrow{\beta} I + I, \quad I \xrightarrow{\gamma} S$$

These interactions provide a master equation, approximated by a Fokker-Planck equation, to give an SDE for disease prevalence. Using Ito's change of variable formula we then derive analytic equations for other data types.

Data Types

Prevalence: $x = I/N$

Rate of Incidence: $\lambda = \beta SI/N$

Incidence: $\nu \approx \lambda\delta$, where δ is the length of time over which incidence data is aggregated.

Equation-Free Method [3]

This is a way of **approximating the potential** surface for a stochastic system **from data**.

For a time-series modelled by an SDE of the form

$$dX = F(X)\,dt + D(X)\,dW$$

We can approximate the drift function at a particular point by

$$F(x) = \lim_{dt \to 0} \left\langle \frac{x(t+dt) - x(t)}{dt} \right\rangle$$

where $\langle \cdot \rangle$ denotes an average. By taking this average over enough data, so that the time-series suitably explores the domain, the entire drift function can be approximated. The potential surface can then be obtained by numerically integrating $-F(x)$.

Results

- Analytic equations for the potential surfaces and results from the equation free method **matched well** for all data types, as shown in the figure on the right.
- **Evidence of critical slowing down** was seen in all data types for the SIS model, giving information about the general behaviour of early warning signals.
- This method could be extended to study early warning signals in more complex models.