Investigating the Potential of Early Warning Signals of Disease Elimination

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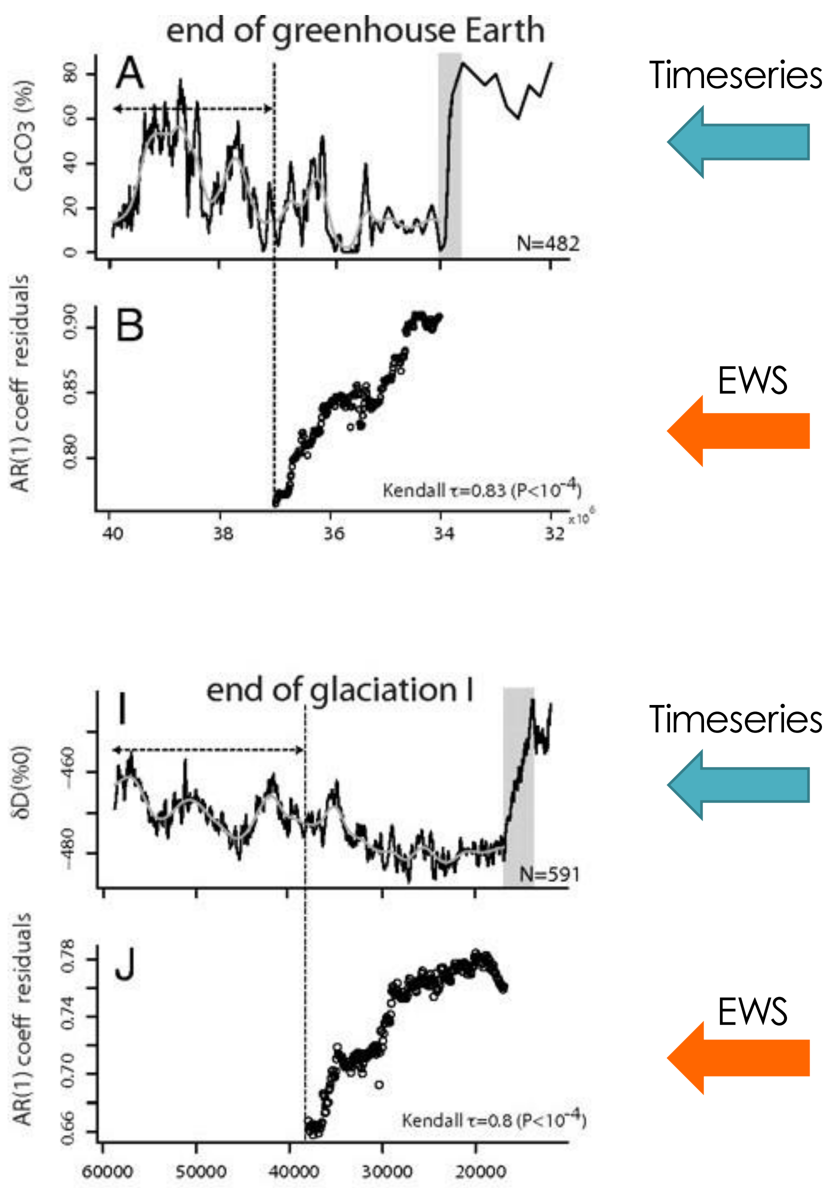
Early Warning Signals (EWS)

- Aim to give early warning of impending **critical transitions** e.g.
  - Abrupt climate change
  - Financial crashes
  - Disease emergence/elimination

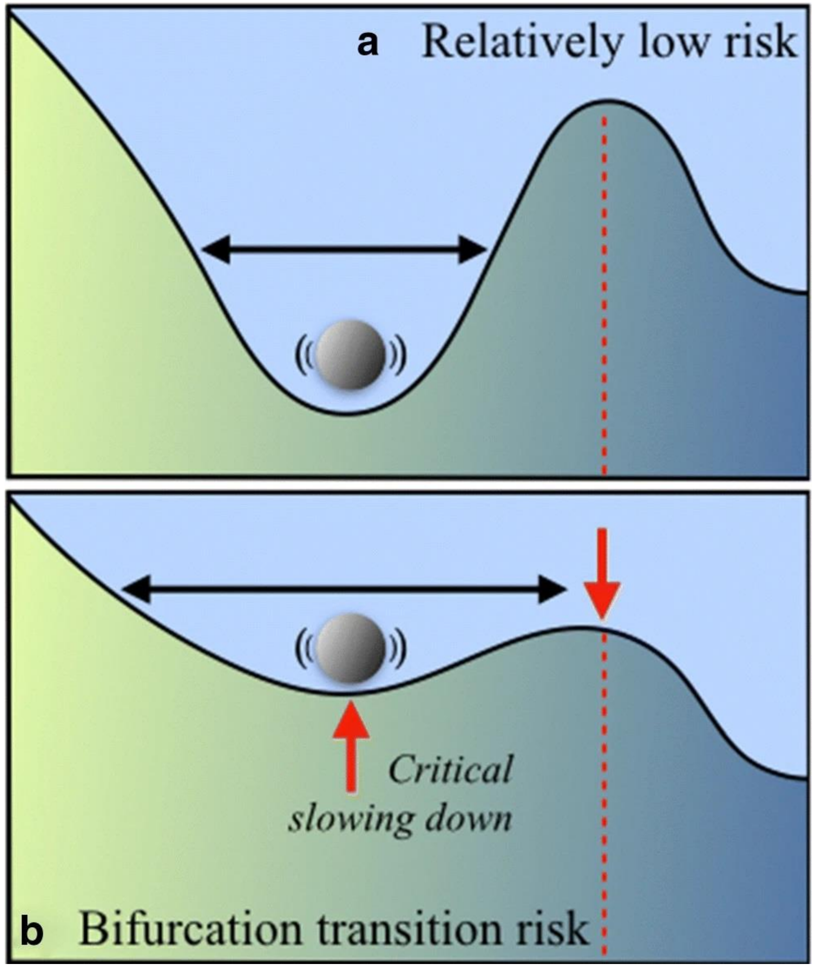
- EWS are statistics calculated on the timeseries e.g.
  - Variance of fluctuations
  - Lag-1 autocorrelation



Adapted from: Dakos *et al.* 'Slowing down as an early warning signal for abrupt climate change'

Critical Slowing Down (CSD)

- Complex systems experience stochastic perturbations.
- **Slower return to equilibrium** following these small disturbances.
- The **potential surface becomes shallower**.



Adapted from: Titus & Watson 'Critical speeding up as an early warning signal of stochastic regime shifts'

Uses within epidemiology

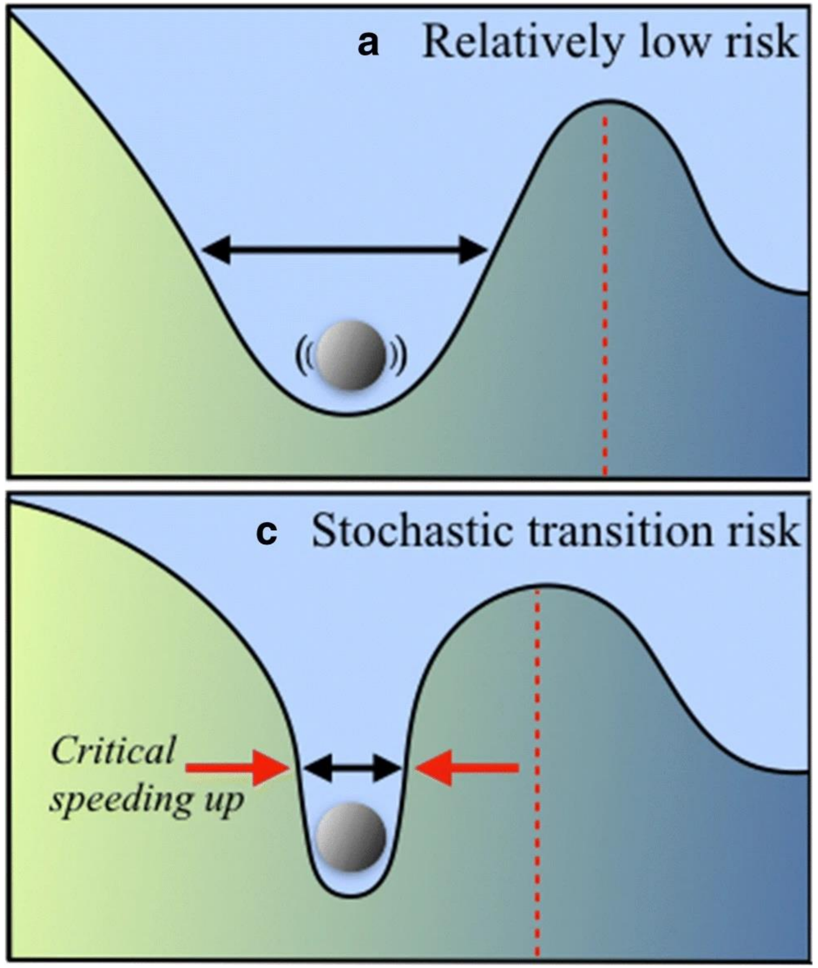
- EWS for disease **emergence**:
  - Early detection of COVID-19 outbreaks within countries.
  - Detecting malarial resurgence.
- EWS for disease **elimination**:
  - Timing the end of control measures.
  - Detecting vaccination herd immunity thresholds.

Implementation Challenges

1. Accurate Calculation
2. Choosing effective EWS
3. Influence from other sources
4. Conflicting theory: Critical Speeding Up

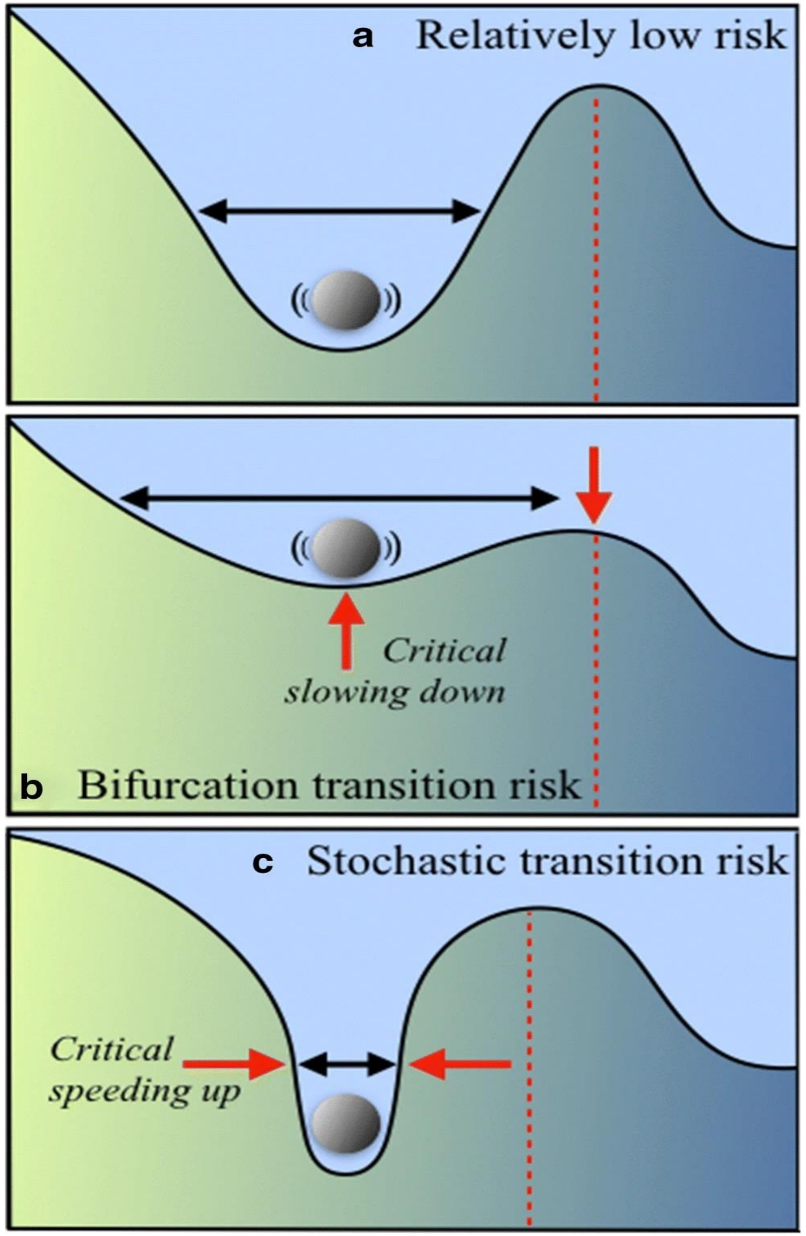
Critical Speeding Up (CSU)

- **Potential surface becomes narrower** but does not change depth.
- **Return to equilibrium becomes faster** approaching the critical transition.
- Variance in fluctuations and autocorrelation both decrease.



The Potential Surface

- Can detect CSD or CSU in a system **without calculating specific EWS**.
- Can be found from ODE or SDE models.
- Offers a **new route** to analysing EWS.



Project Goals

1. Derive **analytic equations for the potential surfaces** for three data types in an SIS model.

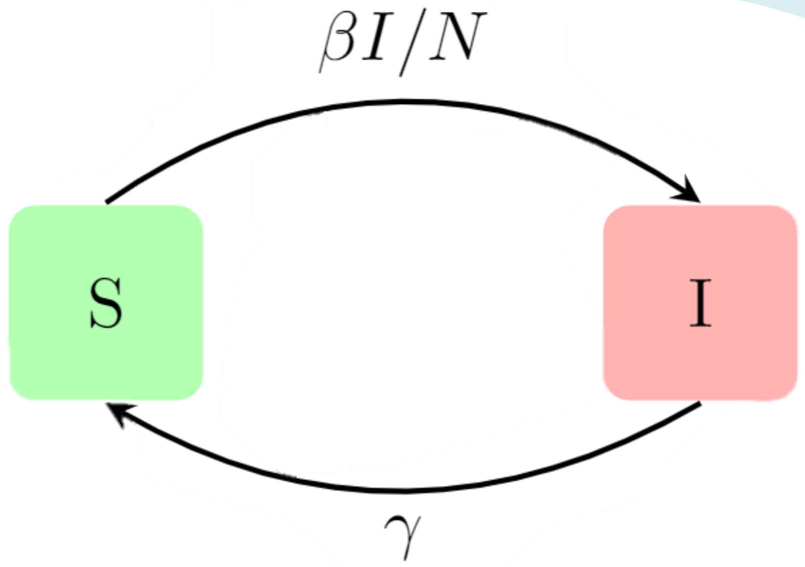
2. Test a method for **approximating the potential surface**.

3. Determine the presence of CSD or CSU in the behaviour of each data type by **analysing the potential surfaces**.

4. **Judge the applicability** of such methods to analysing EWS in other systems.

SIS Model

- Basic reproductive rate $R_0 = \frac{\beta}{\gamma}$
- $R_0 > 1$: Endemic equilibrium is stable, disease free state is unstable.
- $R_0 < 1$: Disease free state stable.
- Critical transition at $R_0 = 1$.



Analytic Method

- Describe prevalence as a stochastic differential equation:

$$dX = F(X)dt + D(X)dB_t \approx {dX}/{dt} = F(X) + \xi_t$$

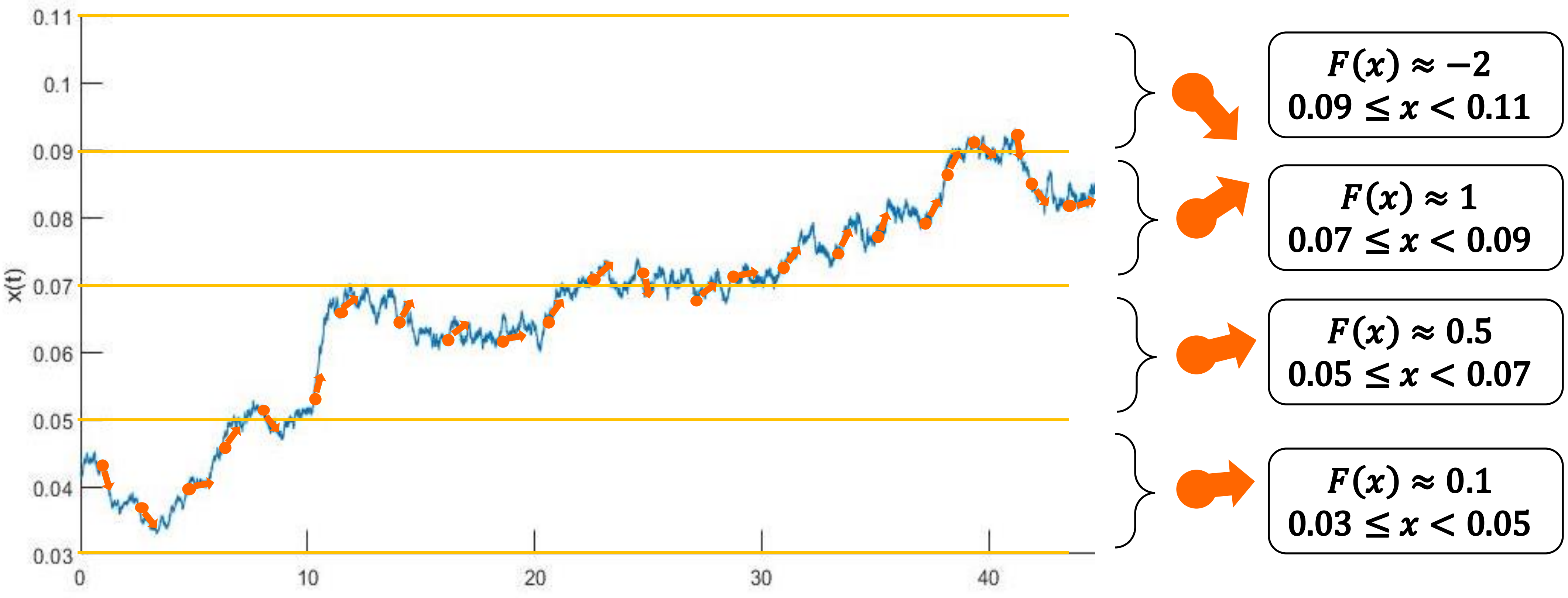
- With potential surface:

$$V(x) = -\int_0^x F(t)dt$$

- Three data types:
  - Prevalence: $\mathrm{x} = I/N$
  - Rate of Incidence (ROI): $\lambda = \beta SI/N$
  - Incidence: $\nu \approx \lambda\delta$

Equation Free Method (EFM)

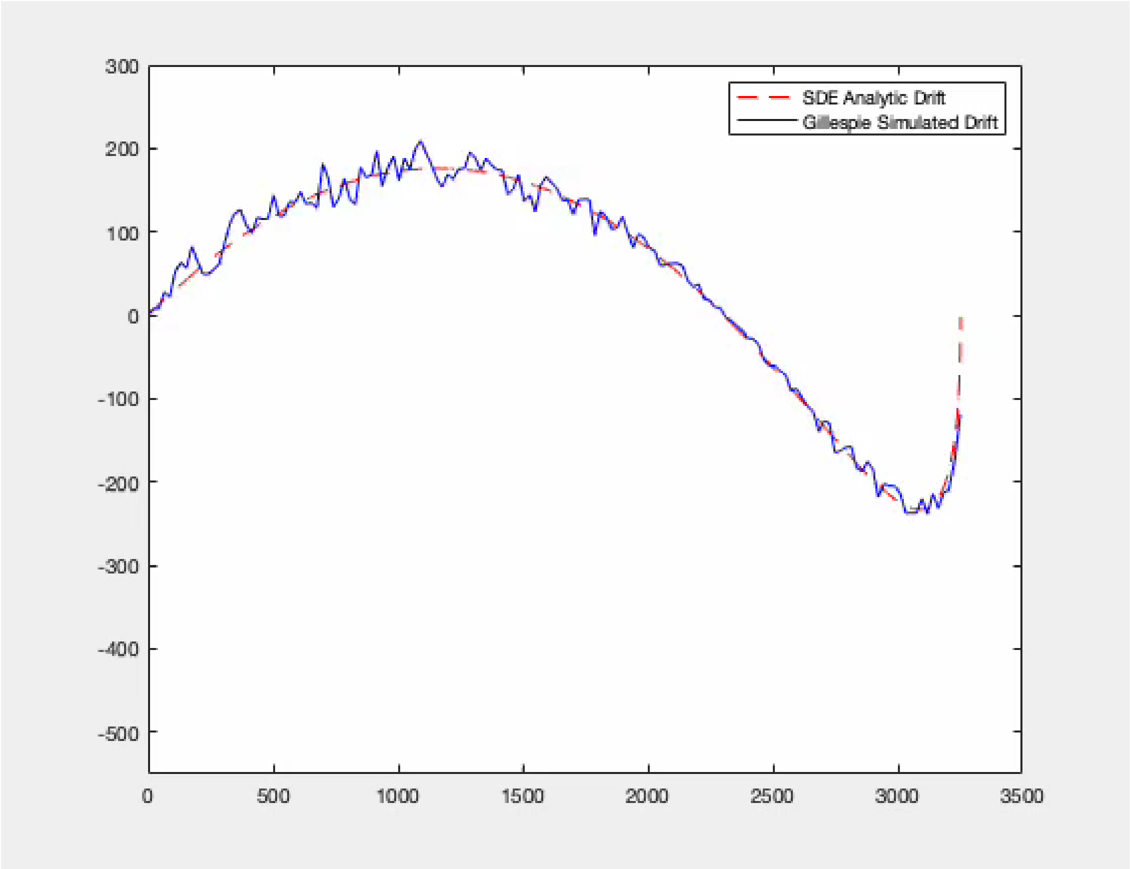
Simulated prevalence, incidence and rate of incidence data is generated using an adapted Gillespie algorithm.

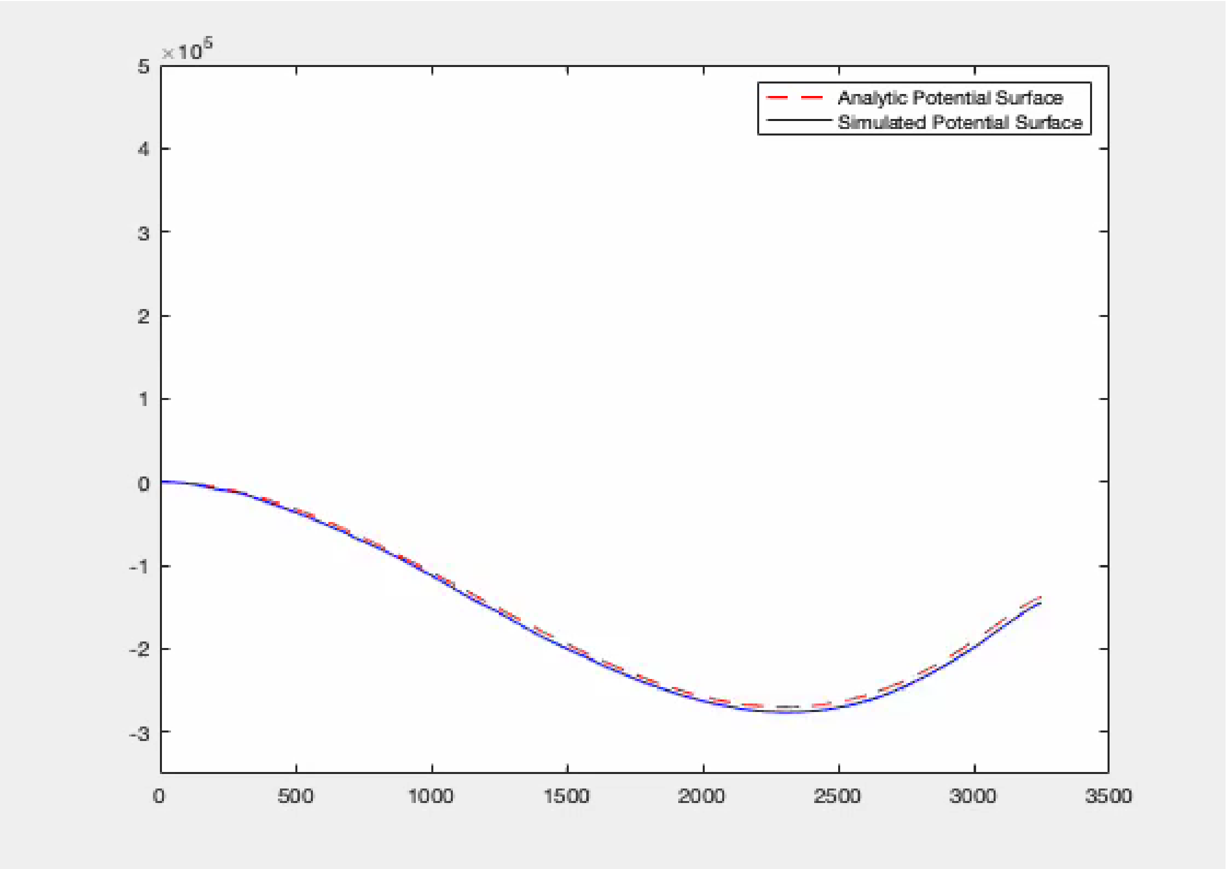


EFM – Sample Results for ROI

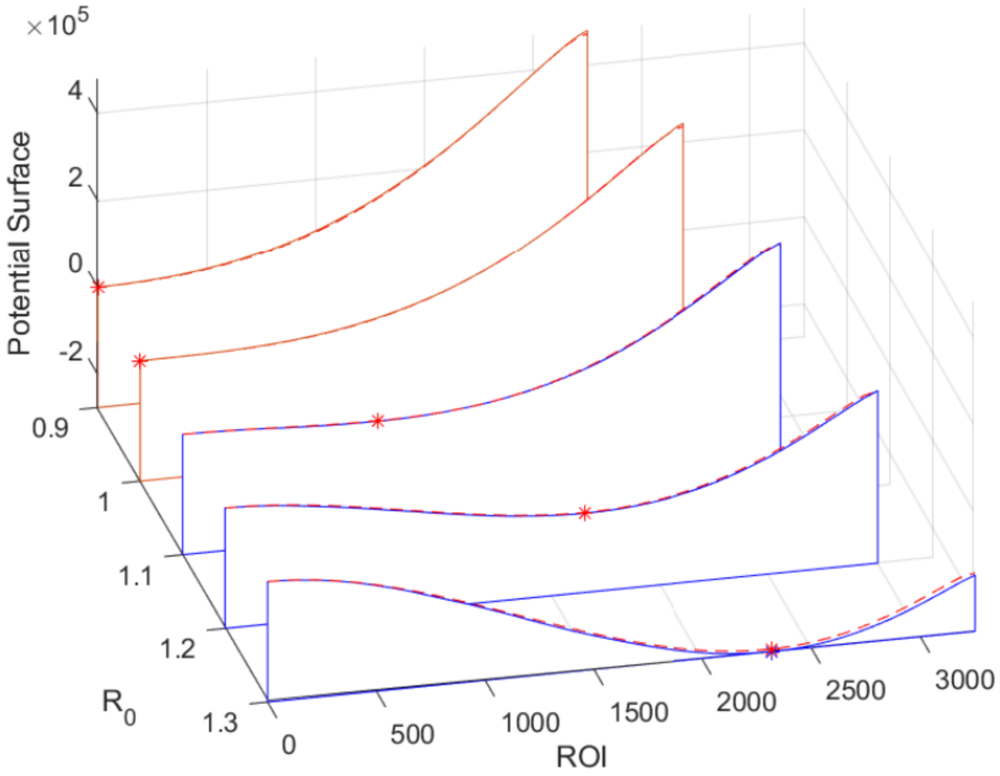
Drift function: $\frac{dx}{dt} = F(x) + \xi_t$

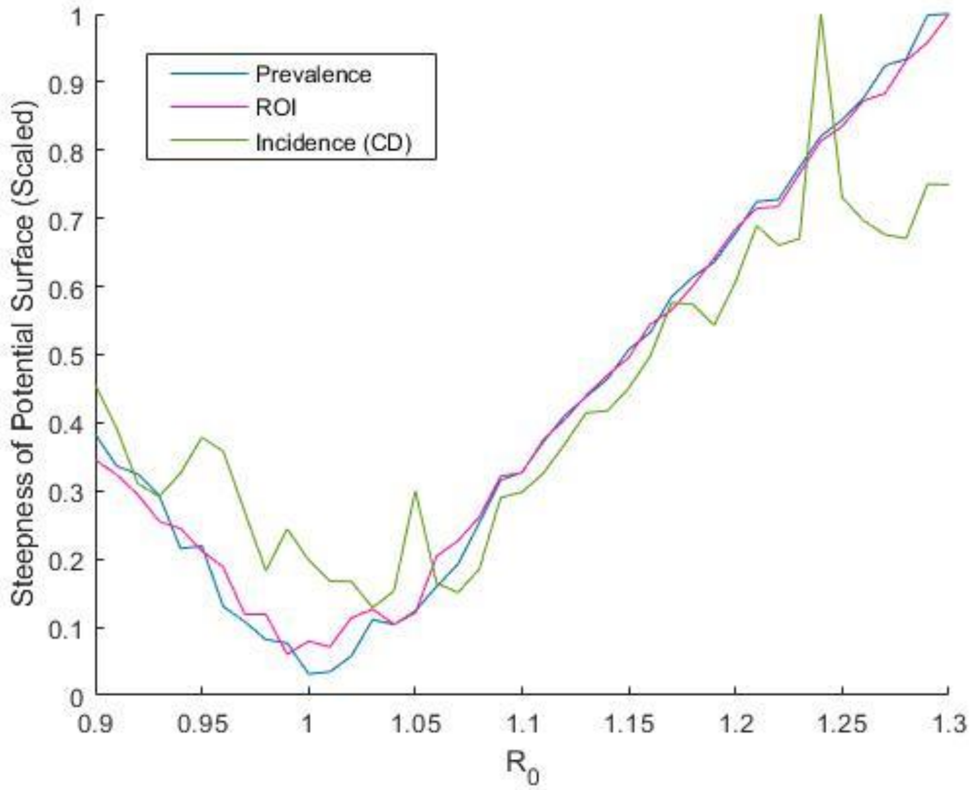
Potential surface: $V(x) = -\int F(x)\,dx$





Results – Rate of Incidence





Conclusions

1. There is clear agreement between the Equation Free Method and analytic results.
2. A flattening of the potential surface was detected for all three data types.
3. Knowing if a system exhibits CSD/CSU is useful, but not necessarily sufficient, to understanding the behaviour of all individual EWS.

Further Work

- Extending the EFM to models in higher dimensions.
- Application of the EFM to test for CSD/CSU in other systems where:
  - CSU is known to occur.
  - Other types of bifurcations occur.
  - EWS have provided inconsistent results.
- Develop the computational efficiency of the EFM (so I can make more nice videos).